Feb. 27, 2013

Announcements:
- X-hour tomorrow
- Optional assignment
- HW7 Due Friday
- WeBWork: 7.8 Due Friday

OH. 1:35-2:35 today
Normal times Thurs.

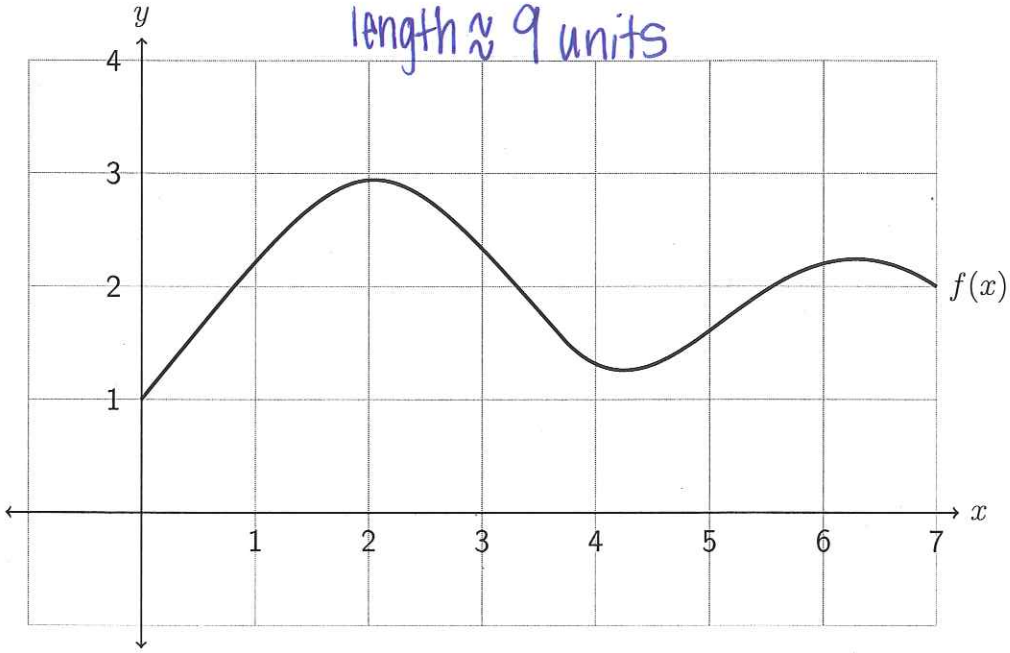
Arc Length (i.e. The Length of a Curve)

- We have been thinking in terms of area under the graph, here is an application that doesn't involve area.
- Arc Length worksheet.

The Length of a Curve

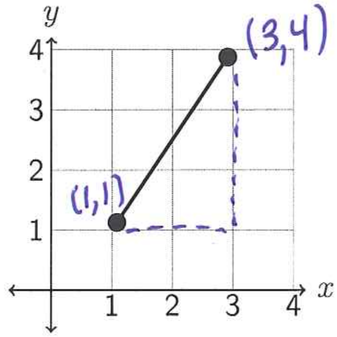
1. Using a string, determine (to the best of your abilities) the length of $f(x)$ between $x = 0$ and $x = 7$. The measurements should be so that one unit is one unit on the graph below (use the x-axis as a ruler).

length $\approx$ 9 units



Unfortunately, when calculating the length of a curve, we don't always have the luxury of using a string and ruler. We want to be able to calculate arc length theoretically and have the same method for *all types of curves.*

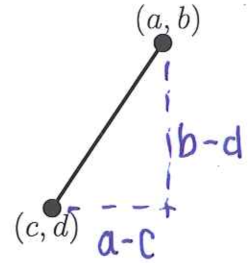
2. (a) What is the length of the line below? What technique(s) did you use to determine the length? What information about the line did you need to figure out?



$\sqrt{2^2+3^2} = \sqrt{13}$

- pythagorean theorem
- need the coordinates of the endpoints.

(b) What is the length of the line below, given the coordinates of its endpoints are (a, b) and (c, d) (as shown)?

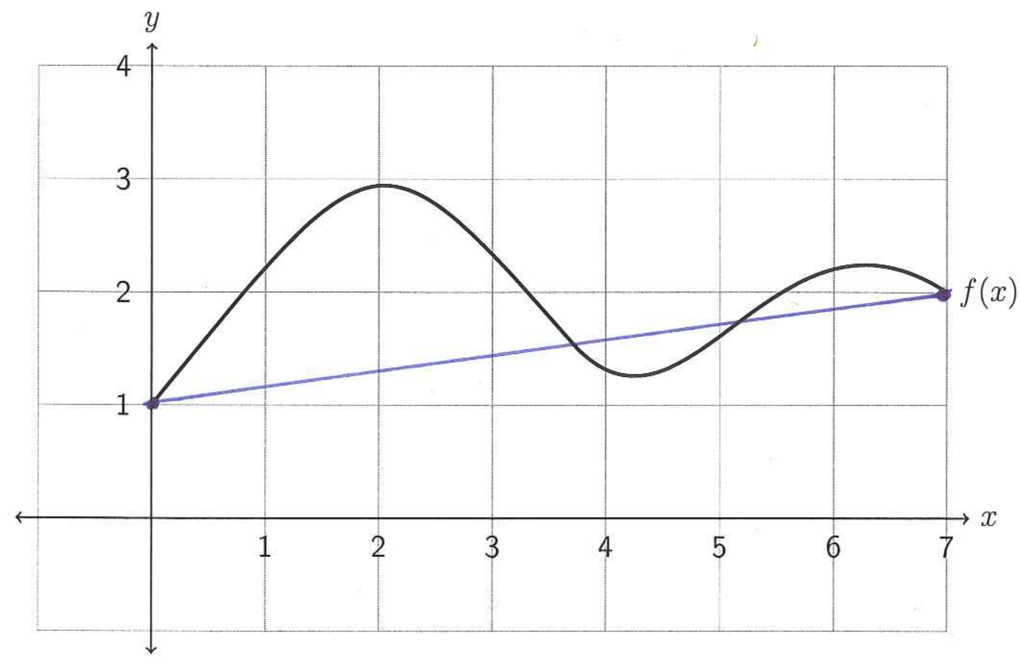


$\sqrt{(a-c)^2+(b-d)^2} = \text{length}$

3. Problem 2 tells us something very important: we can relatively easily determine the length of a stright line, provided we know the endpoints. Discuss with your partner: How could you use this fact to estimate the length of a curve. You should have a fairly clear idea on how to implement your idea and convey it to another person. When you think you have a sound idea, **explain it to your professor before proceeding.**

4. Estimate the length of the curve $f(x)$ using the numbers of lines listed below. (Use the graph to approximate the values of the endpoints of your lines).

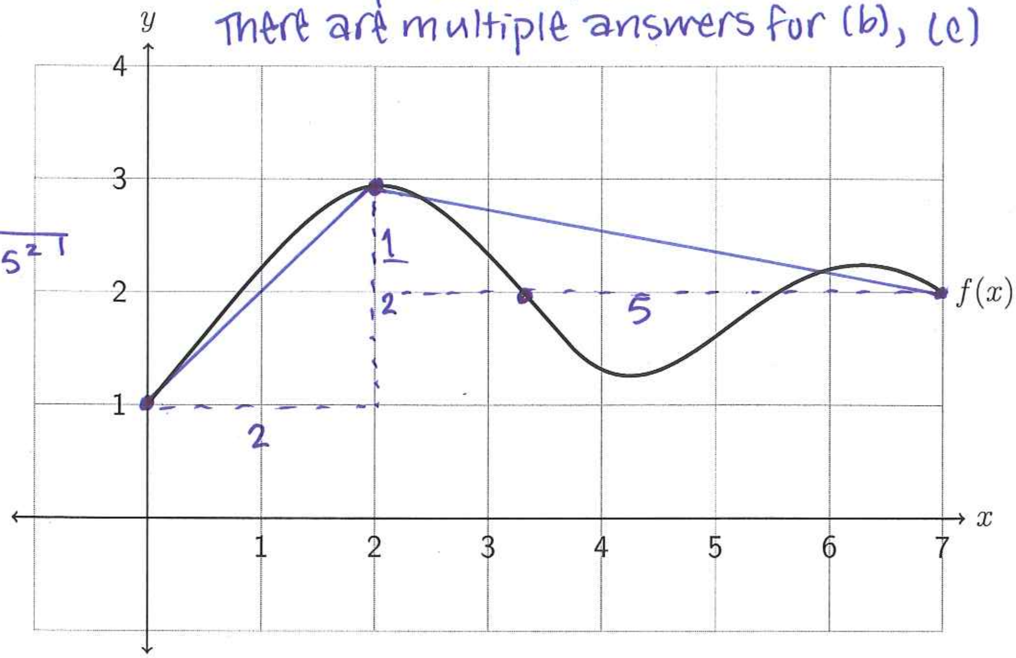
(a) One line.



Approx $= \sqrt{1^2 + 7^2} = \sqrt{50} \approx 7.07$

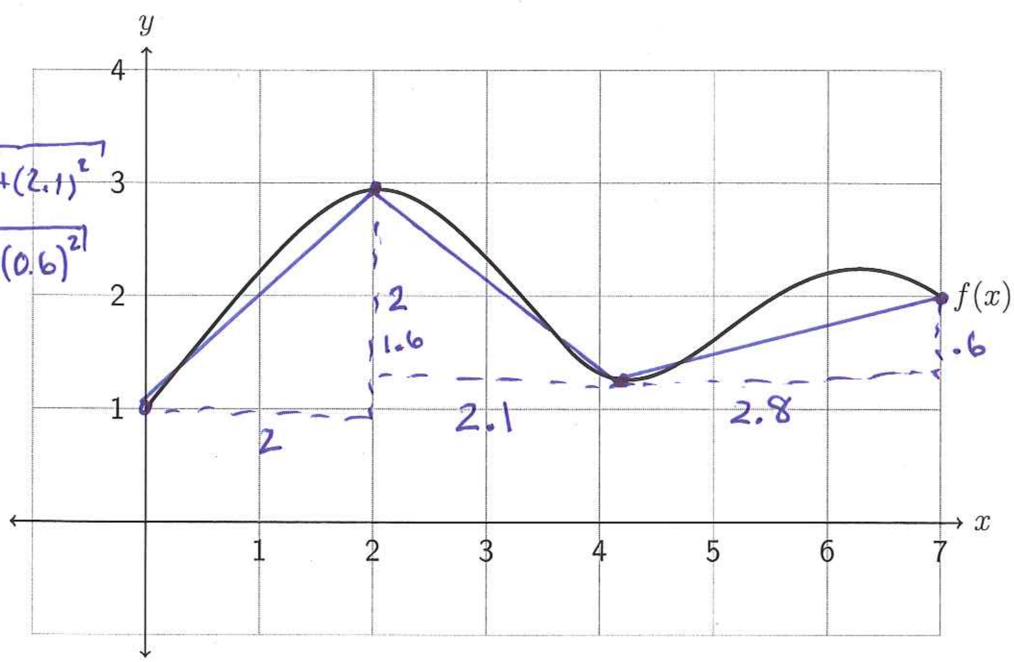
(b) Two lines.

Wherever you connect the lines is fine.
There are multiple answers for (b), (c)



$$\sqrt{2^2+2^2}+\sqrt{1+5^2}$$
$$=\sqrt{8}+\sqrt{26}$$
$$\approx 7.927$$

(c) Three lines.

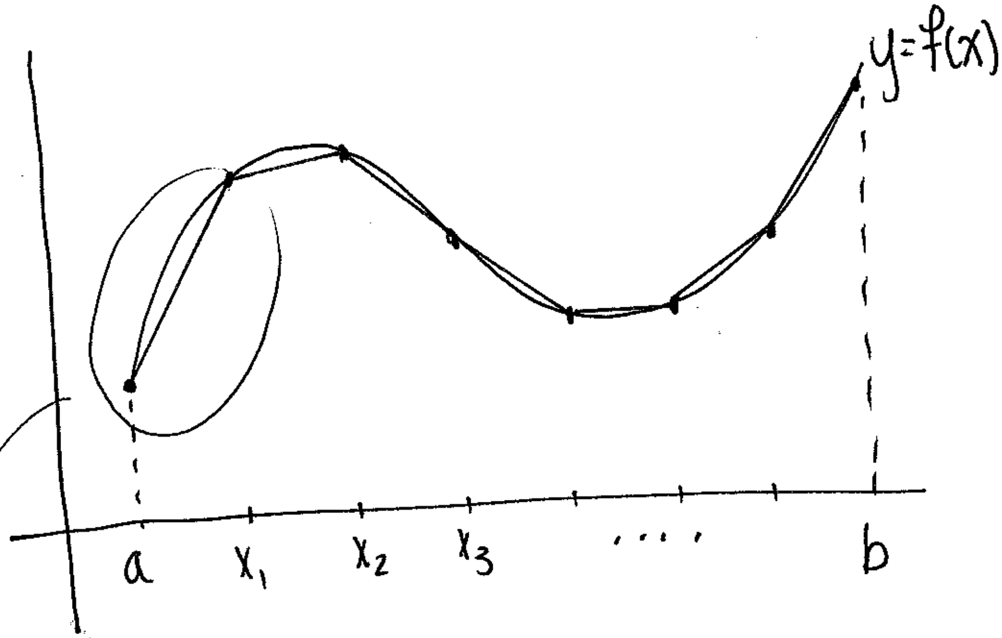


$$\sqrt{2^2+2^2}+\sqrt{(1.6)^2+(2.1)^2}$$
$$+\sqrt{(2.8)^2+(0.6)^2}$$
$$=8.33$$

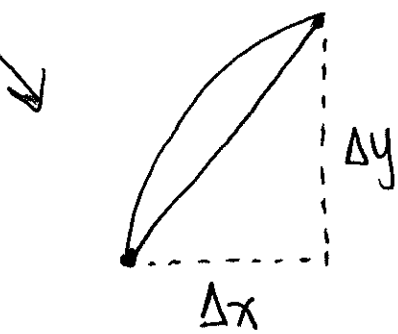
Which estimate is closer to the actual length of the curve? What changes can you make to your technique to more closely approximate the length of the curve?

The Arc Length "Intuition"

The more lines we use to estimate arc length, the closer to the real arc length we will become.



zoom in



hypotenuse $= \sqrt{(\Delta x)^2 + (\Delta y)^2}$

The not completely rigorous way to get Arc Length

$\Delta x \approx dx \qquad \Delta y \approx dy$

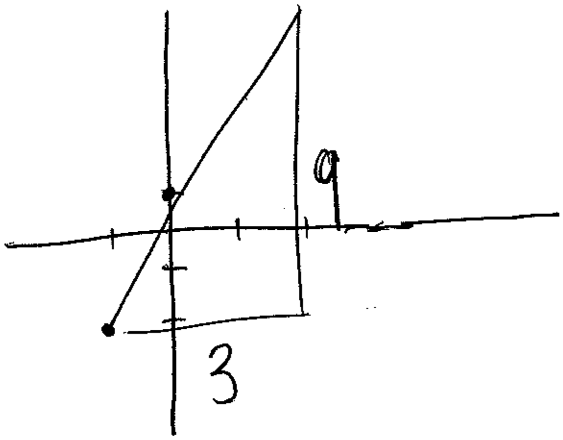
So $\sqrt{(\Delta x)^2 + (\Delta y)^2} \approx \sqrt{(dx)^2 + (dy)^2} \approx \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$$\frac{dy}{dx} = f'(x)$$

Arc Length Formula: for $f(x)$ between $x=a$, $x=b$

$$\int_a^b \sqrt{1+(f'(x))^2}\,dx$$

<u>Example</u>: length of $y=3x+1$, between $x=-1$, $x=2$



$$\sqrt{3^2+9^2} = \sqrt{90} = 3\sqrt{10}$$

Arc length formula:

$$\int_{-1}^{2} \sqrt{1+\left(\frac{d}{dx}(3x+1)\right)^2}\,dx = \int_{-1}^{2} \sqrt{1+9}\,dx$$

$$= \int_{-1}^{2} \sqrt{10}\,dx = 2\sqrt{10}+1\sqrt{10}$$

$$= \boxed{3\sqrt{10}}$$

equal. ✓