Feb. 18, 2013

Announcements: X-hour midterm Review
Midterm 2 on Thurs (7-9, Wilder 111)
WeBWork: 7.3 Due Wed, 7.4 Due Mon.
HW6 Due Mon.

Trig Sub:

Last Time you did: $\int \frac{x^3}{\sqrt{x^2-1}}\,dx$, $\int \frac{x-1}{\sqrt{9-x^2}}\,dx$

Remember Subbing Table:

| Expression | What to Sub | Identity |
| --- | --- | --- |
| $\sqrt{a^2-x^2}$ | $x = a\sin\theta$ | $1-\sin^2\theta = \cos^2\theta$ |
| $\sqrt{a^2+x^2}$ | $x = a\tan\theta$ | $1+\tan^2\theta = \sec^2\theta$ |
| $\sqrt{x^2-a^2}$ | $x = a\sec\theta$ | $\sec^2\theta - 1 = \tan^2\theta$ |

*Word of Caution: Don't use trig sub if you can use regular substitution.*

**Example:** $\displaystyle\int \frac{x\,dx}{\sqrt{1+x^2}}$

* use u-substitution *

$u = 1+x^2,\ du = 2x\,dx$

$$\int \frac{x}{\sqrt{1+x^2}}\,dx = \int \frac{1}{2}\,\frac{1}{\sqrt{u}}\,du = \frac{1}{2}\cdot 2\sqrt{u} + C = \sqrt{u} + C = \sqrt{1+x^2} + C$$

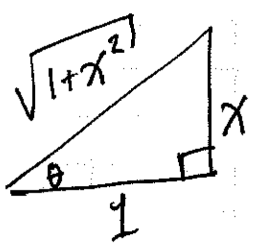
* trig sub *

$x = \tan\theta$
$dx = \sec^2\theta\,d\theta$

$$\int \frac{x\,dx}{\sqrt{1+x^2}} = \int \frac{\tan\theta\cdot\sec^2\theta\,d\theta}{\sqrt{1+\tan^2\theta}}$$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{x}{1}$

$$= \int \frac{\tan\theta\sec^2\theta}{\sqrt{\sec^2\theta}}\,d\theta = \int \tan\theta\sec\theta\,d\theta$$




$$= \sec\theta + C$$
$$= \sqrt{1+x^2} + C$$

**Definite integrals:** Plug in the bounds at the end.

**Example:** $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$

• Note that we can't use u-sub.

$x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

we get: $\displaystyle\int \frac{1}{4\tan^2\theta\sqrt{4\tan^2\theta+4}}\cdot 2\sec^2\theta\,d\theta$

$$= \int \frac{2\sec^2\theta\,d\theta}{4\tan^2\theta\sqrt{4\sec^2\theta}} = \int \frac{\sec^2\theta}{2\tan^2\theta\cdot 2\sec\theta}\,d\theta$$

$$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{4}\int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\,d\theta$$

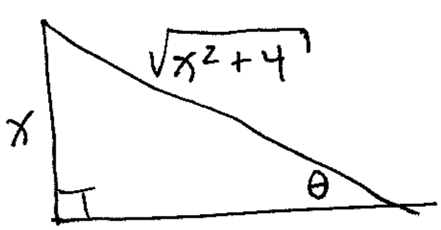
$$= \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

use u-sub: $u = \sin\theta$
$du = \cos\theta\,d\theta$

we get: $\displaystyle\frac{1}{4}\int \frac{1}{u^2}\,du = \frac{1}{4}\left(-\frac{1}{u}\right) + C$

$$= -\frac{1}{4}\cdot\frac{1}{\sin\theta} + C$$ sub back in for u

sub x back in:




$x = 2\tan\theta \Rightarrow \tan\theta = \frac{x}{2} = \frac{opp}{adj}$

so:
$\sin\theta = \frac{x}{\sqrt{x^2+4}}$

$$-\frac{1}{4}\cdot\frac{1}{\sin\theta} + C = \boxed{-\frac{1}{4}\cdot\frac{\sqrt{x^2+4}}{x} + C}$$

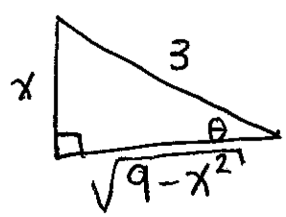
# Practice Problems:

1. $$\int \frac{dx}{x^2\sqrt{9-x^2}}$$ $\qquad x = 3\sin\theta$, $\quad dx = 3\cos\theta\, d\theta$

$$= \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta\sqrt{9(1-\sin^2\theta)}}$$

$$= \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta\sqrt{9\cos^2\theta}} = \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta \cdot 3\cos\theta} = \int \frac{1}{9}\cdot \csc^2\theta\, d\theta = \frac{-1}{9}\cot\theta + C$$

Sub back in:




$x = 3\sin\theta \Rightarrow \frac{x}{3} = \sin\theta$

so $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{9-x^2}}{x}$

$$-\frac{1}{9}\cot\theta + C = \boxed{\frac{-1}{9}\cdot\frac{\sqrt{9-x^2}}{x} + C}$$

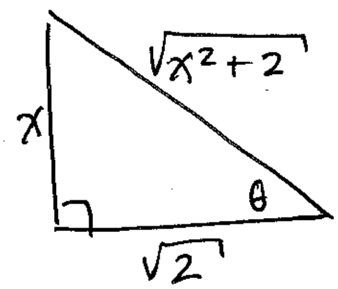
2. $$\int_0^2 \frac{x^3}{\sqrt{x^2+2}}\,dx$$ $\qquad x = \sqrt{2}\tan\theta$, $\quad dx = \sqrt{2}\sec^2\theta\, d\theta$

$$= \int_{x=0}^{x=2} \frac{2\sqrt{2}\cdot\tan^3\theta\cdot\sqrt{2}\sec^2\theta\, d\theta}{\sqrt{2\tan^2\theta+2}} = \int_{x=0}^{x=2}\frac{4\tan^3\theta\sec^2\theta\, d\theta}{\sqrt{2}\sec\theta} = \frac{4}{\sqrt{2}}\int_{x=0}^{x=2}\tan^3\theta\sec\theta\, d\theta$$

$$= \frac{4}{\sqrt{2}}\int_{x=0}^{x=2}(\sec^2\theta-1)\tan\theta\sec\theta\, d\theta$$ $\qquad u = \sec\theta$, $\quad du = \sec\theta\tan\theta\, d\theta$

$$= \frac{4}{\sqrt{2}}\int_{x=0}^{x=2}(u^2-1)\,du = \frac{4}{\sqrt{2}}\left(\frac{u^3}{3}-u\right)\Bigg|_{x=0}^{x=2} = \frac{4}{\sqrt{2}}\left(\frac{\sec^3\theta}{3}-\sec\theta\right)\Bigg|_{x=0}^{x=2}$$

*Get in terms of $x$ *




$$x=\sqrt{2}\tan\theta \implies \frac{x}{\sqrt{2}}=\tan\theta$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{x^2+2}}{\sqrt{2}}$$

$$\frac{4}{\sqrt{2}}\left(\frac{\sec^3\theta}{3}-\sec\theta\right)\Bigg|_{x=0}^{x=2} = \frac{4}{\sqrt{2}}\left(\frac{1}{3}\left(\frac{\sqrt{x^2+2}}{\sqrt{2}}\right)^3-\frac{\sqrt{x^2+2}}{\sqrt{2}}\right)\Bigg|_1^2$$

$$= \frac{(x^2-4)\sqrt{x^2+2}}{3}\Bigg|_1^2$$

$$= 0 - \frac{(-3)\sqrt{3}}{3} = \sqrt{3}$$

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3. $$\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}\,dt = \qquad t=\sec\theta \quad dt=\sec\theta\tan\theta\,d\theta$$

$$= \int_{t=\sqrt{2}}^{t=2} \frac{1}{\sec^3\theta\sqrt{\sec^2\theta-1}}\cdot\sec\theta\tan\theta\,d\theta$$

$$= \int_{t=\sqrt{2}}^{t=2} \frac{\sec\theta\tan\theta\,d\theta}{\sec^3\theta\tan\theta} = \int_{t=\sqrt{2}}^{t=2}\frac{1}{\sec^2\theta}\,d\theta = \int_{t=\sqrt{2}}^{t=2}\cos^2\theta\,d\theta$$

3. $\displaystyle\int_4^5 \frac{dt}{t^2\sqrt{t^2-16}}$ $\qquad t = 4\sec\theta$

$\qquad dt = 4\sec\theta\tan\theta\, d\theta$

$$= \int_{t=4}^{t=5} \frac{4\sec\theta\tan\theta\,d\theta}{16\sec^2\theta\sqrt{16\sec^2\theta-16}} = \int_{t=4}^{t=5}\frac{\sec\theta\tan\theta}{16\sec^2\theta\cdot\tan\theta}\,d\theta = \frac{1}{16}\int_{t=4}^{t=5}\frac{1}{\sec\theta}\,d\theta$$

$$= \frac{1}{16}\int_{t=4}^{t=5}\cos\theta\,d\theta = \frac{\sin\theta}{16}\Big|_4^5 = \frac{\sqrt{t^2-16}}{16t}\Big|_4^5 = \frac{3}{16\cdot 5} - 0 = \boxed{\frac{3}{80}}$$

$\sec\theta = \frac{t}{4} = \frac{hyp}{adj}$