Feb. 1, 2013

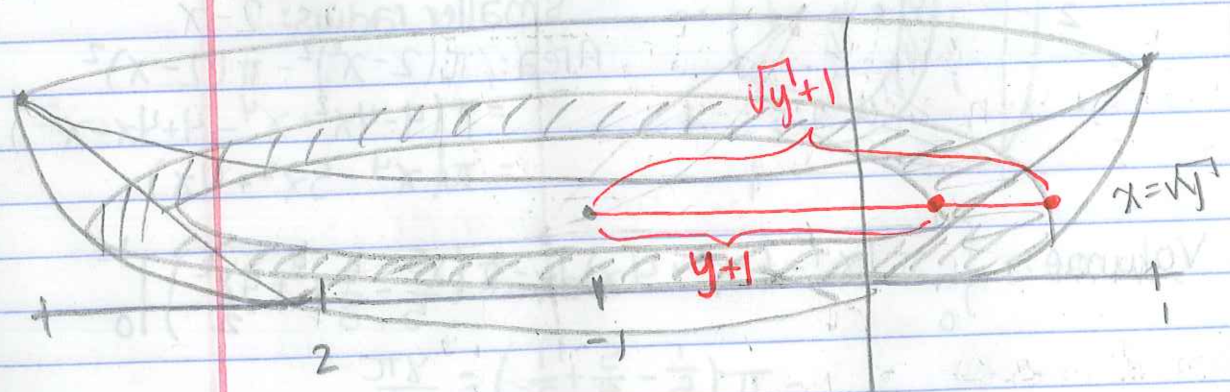
Announcements: HW3 Due today (get trucking on HW4)
Webwork: 6.2 Volumes due Mon
6.3 Cylindrical Shells due Wed.

Finding the volume of a Solid of Revolution w/ slices

Things to-do:

- Decide how you will slice your solid.
  - If slicing perpendicular to x-axis your integral will be in terms of x
  - If slicing perpendicular to y-axis your integral will be in terms of y.
- Find the area of your slices.
  - Disk: $A = \pi(\text{radius})^2$
  - Washer: $A = \pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$

Practice (6) Volume of solid obtained by rotating region enclosed by $y=x$, $y=x^2$ about the line $x=-1$.



outer radius $= \sqrt{y}+1$

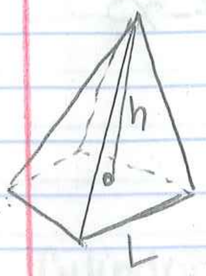
inner radius $= y+1$

$$\int_0^1 \pi\left((\sqrt{y}+1)^2-(y+1)^2\right)dy = \int_0^1 \pi\left(y+2\sqrt{y}+1-y^2-2y-1\right)dy$$

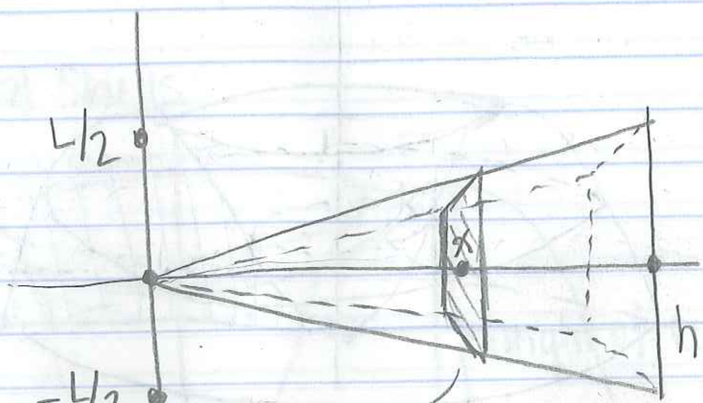
$$= \pi\int_0^1 \left(2\sqrt{y}-y-y^2\right)dy = \pi\left(\frac{4}{3}y^{3/2}-\frac{y^2}{2}-\frac{y^3}{3}\right)\Big|_0^1$$

$$= \pi\left(\frac{4}{3}-\frac{1}{2}-\frac{1}{3}\right) = \pi/2$$

(7) Find the volume of a pyramid whose base is a square with side of length L and whose height is h.

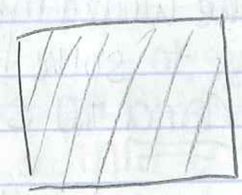


put on coordinate axes to make computations as easy as possible.



cross-section is a square:

Length of cross-sectional square is L at $x=h$ and 0 at $x=0$ and decreases linearly so



length at x is $\frac{L}{h} \cdot x$

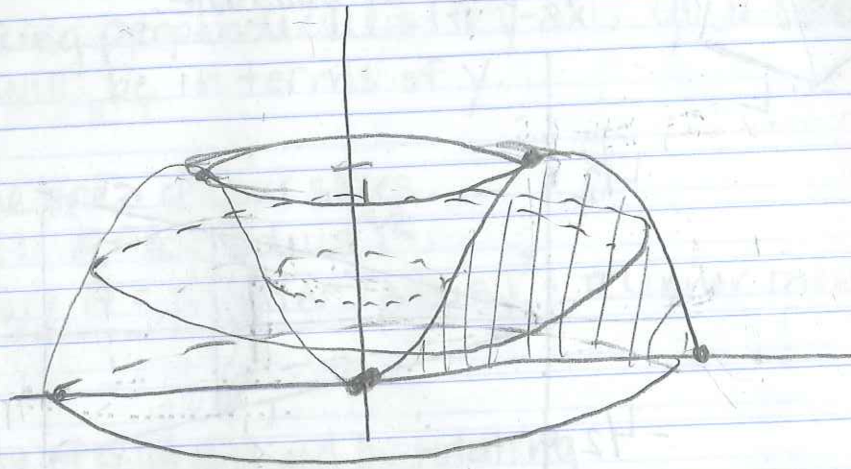
Area of cross-section is $\left(\frac{L}{h} \cdot x\right)^2 = \frac{L^2}{h^2} x^2$

Integrate: $$V = \int_0^h \frac{L^2}{h^2} x^2 \, dx = \underbrace{\frac{L^2}{h^2}}_{\text{constants}} \int_0^h x^2 \, dx$$

$$= \frac{L^2}{h^2} \left(\frac{x^3}{3}\right)\Big|_0^h = \frac{L^2}{h^2} \cdot \frac{h^3}{3} = \boxed{\frac{1}{3} L^2 h}$$

Cylindrical Shells

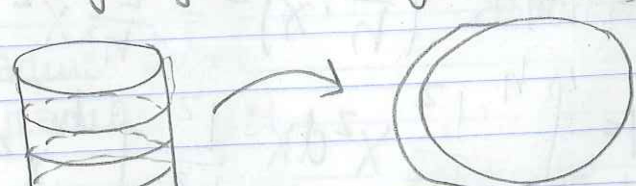
Find the volume of the solid obtained by rotating the region bounded by $y=2x^2-x^3$ and $y=0$ about the y-axis



Slicing:
We would want to slice perpendicular to the y-axis.
Then we would integrate w/ respect to y, so we have to solve for x,
It's <u>hard</u> to solve this equation for x.

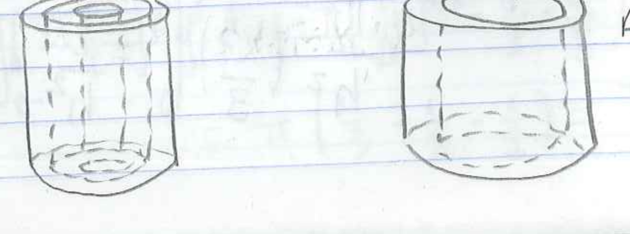
We need a new method.
- instead of slicing our solid into disks or washers we are going cut out cylindrical shells

Slicing:



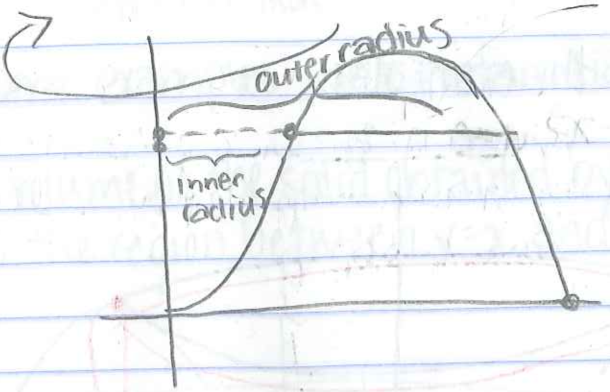
A slice

Cylindrical shells

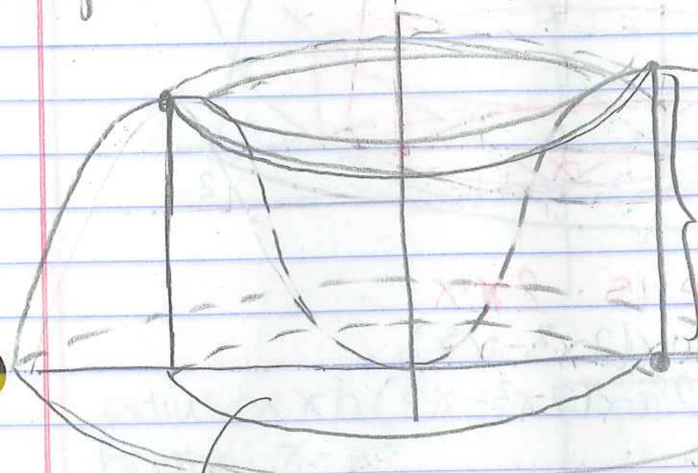


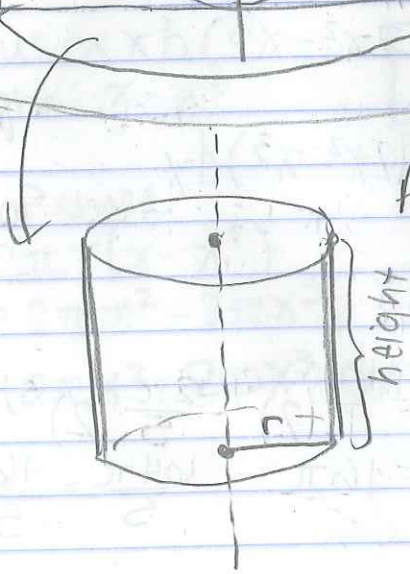
A shell (hollow inside)

Slicing



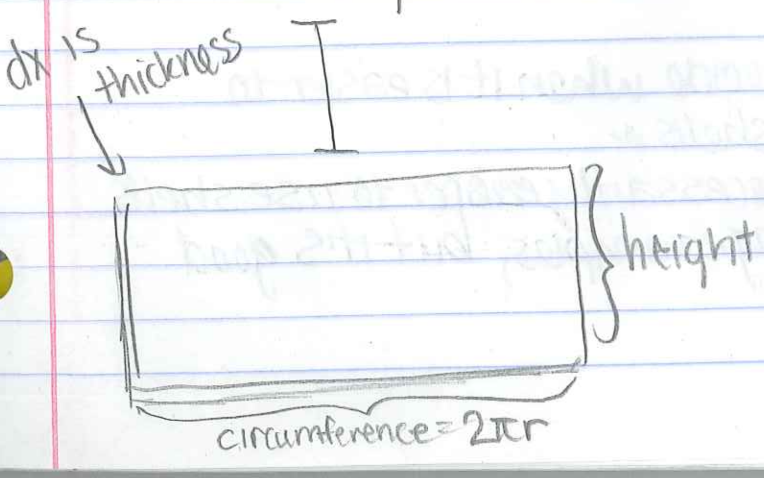
Cylindrical Shells:





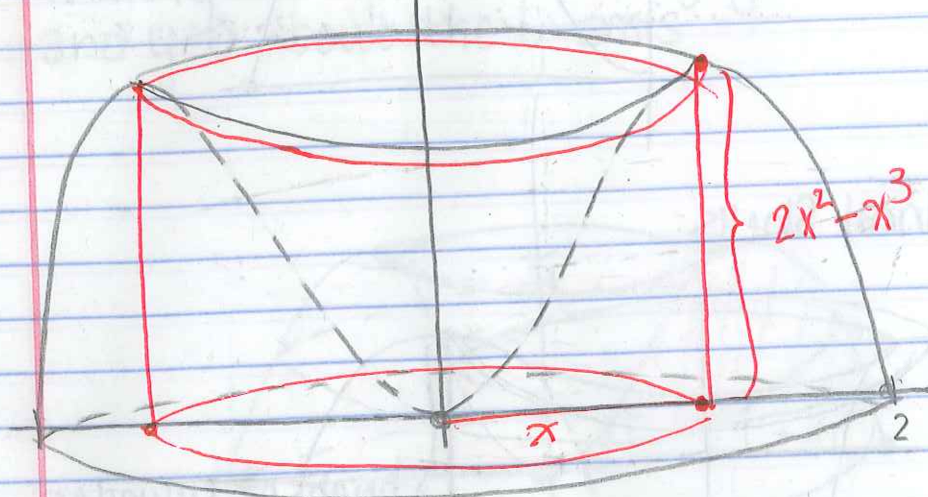
A hollow cylinder w/ infinitely thin walls.

What would the volume of one of these cylinders be?



Back to the example:

$y = 2x^2 - x^3,\ y = 0$



circumference is $2\pi x$

Area of shell: $2\pi x(2x^2 - x^3)$

Volume of shell: $2\pi x(2x^2 - x^3)\,dx$

Volume of solid: $\int_0^2 2\pi x(2x^2 - x^3)\,dx$

$= \int_0^2 4\pi x^3 - 2\pi x^4 \, dx$

$= \pi x^4 - \frac{2\pi}{5}x^5 \Big|_0^2 = \pi(2)^4 - \frac{2\pi}{5}(2)^5$

$= 16\pi - \frac{64\pi}{5} = \frac{16}{5}\pi$

* You need to decide when it is easier to us slices or shells *

It's not necessarily easier to use shells on the following examples, but it's good practice.