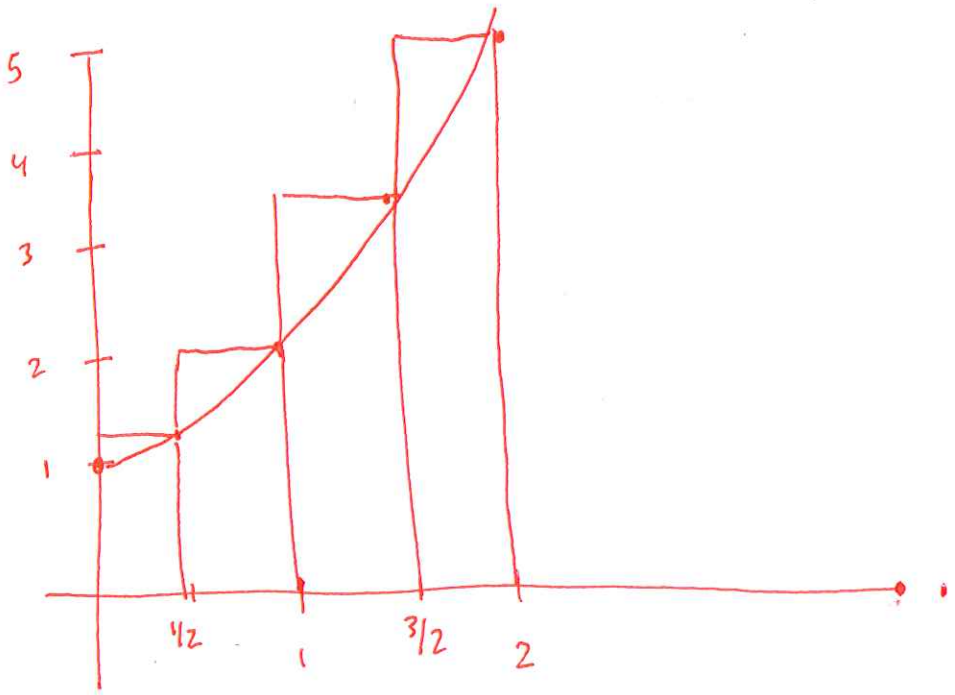
Quiz 1: Area and The Definite Integral

January 16, 2013

Name: Key Section: _____

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. Answer both questions (second one is on the back).

1. Evaluate the Riemann sum for $f(x) = x^2 + 1$ between 0 and 2 using four subintervals, taking the sample points to be the right endpoints of the subintervals. Draw a diagram with the curve $f(x)$ and the four rectangles. Explain what value the Riemann sum approximates.



$$\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$$

$$\text{Area} \approx \frac{1}{2}\left(\left(\tfrac{1}{2}\right)^2+1\right) + \frac{1}{2}\left(1^2+1\right) + \frac{1}{2}\left(\left(\tfrac{3}{2}\right)^2+1\right) + \frac{1}{2}\left(2^2+1\right)$$

The Riemann sum approximates the area below the function $f(x)$ above the x-axis and between the values $x=0$ and $x=2$.

2. Given that $\int_0^\pi \sin^2 x\,dx = \frac{\pi}{2}$ find $\int_0^\pi 4\sin^2 x + 7\,dx$.

$$\int_0^\pi 4\sin^2 x + 7\,dx = \int_0^\pi 4\sin^2 x\,dx + \int_0^\pi 7\,dx$$

$$= 4\int_0^\pi \sin^2 x\,dx + \int_0^\pi 7\,dx = 4\cdot\frac{\pi}{2} + 7(\pi - 0)$$

$$= 9\pi$$