Last time: Arc Length

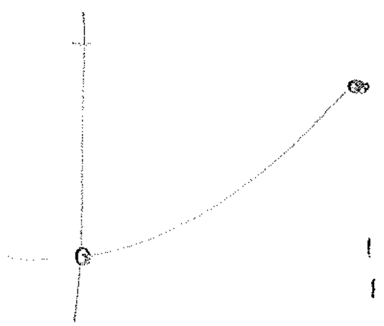
$$\text{length} = \int_a^b \sqrt{1+[f'(x)]^2}\, dx$$

$y' = x^{1/2}$

$(y')^2 = x$

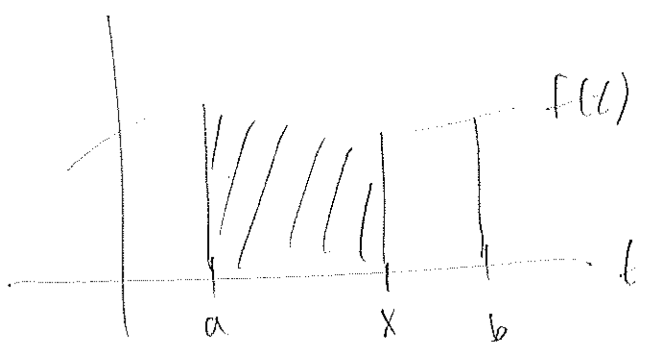
ex) find length of $y = \frac{2}{3}x^{3/2}$, $0 \le x \le 1$

$$\text{length} = \int_0^1 \sqrt{1+x}\, dx = \frac{2}{3}\cdot(1+x)^{3/2}\Big|_0^1 = \frac{2}{3}\cdot(2)^{3/2} - \frac{2}{3} \approx 1.2$$



Arc length Function: # 8.1.35 "arc length so far"

Recall: Area so far $g(x) = \int_a^x f(t)\,dt \quad a \le x \le b$



$$s(x) = \int_a^x \sqrt{1 + [f'(t)]^2}\, dt \qquad a \le x \le b$$

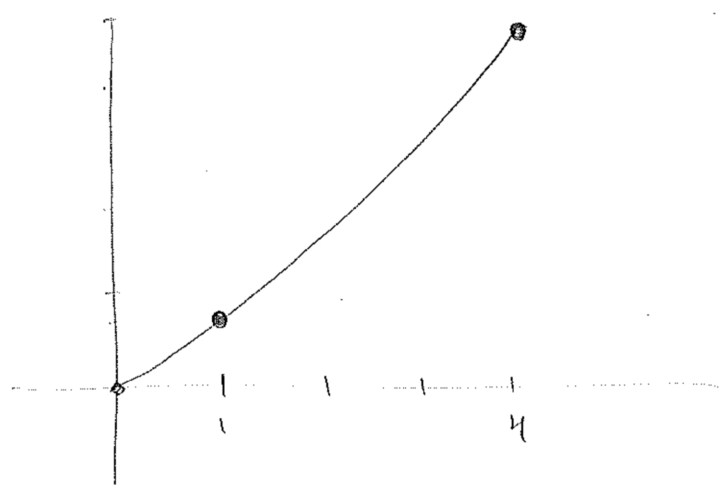
Gives the arc length of the curve f(t) from initial point (a, f(a)) to another point (x, f(x))

ex) find arc length function for curve $y = \frac{2}{3} x^{3/2}$ with starting point $(1, 2/3)$.

$y' = x^{1/2} \qquad (y')^2 = x$

$$s(x) = \int_1^x \sqrt{1+t}\, dt = \frac{2}{3}(1+t)^{3/2} \Big|_1^x = \frac{2}{3}(1+x)^{3/2} - \frac{2}{3} \cdot 2^{3/2}$$

Hence $s(4) = \frac{2}{3}(1+4)^{3/2} - \frac{2}{3} 2^{3/2} \approx 5.6$

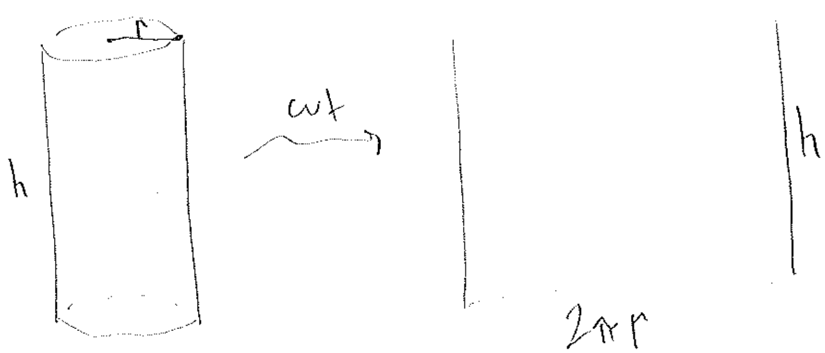


§8.2 Surface Area

def surface of revolution is formed when a curve is rotated about on of the axes. It is the boundary (or shell) of the solids of revolution from 6.2 and 6.3

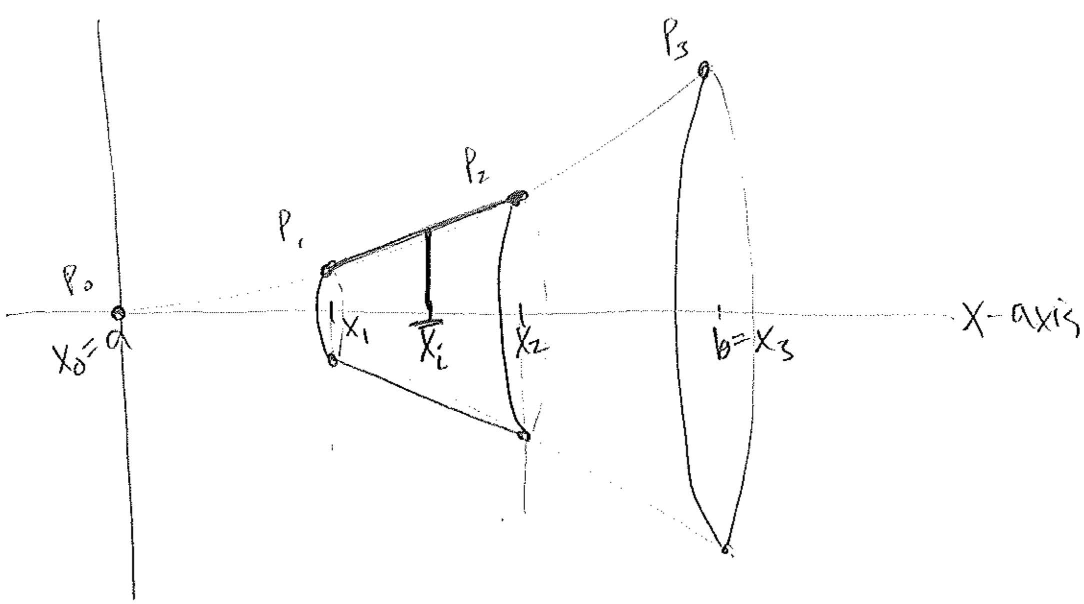
We will be computing the surface area of these surfaces of revolution.

ex cylinder



$SA = 2\pi r \cdot h$

More general surface of revolution:



$$d(P_i, P_{i+1}) = \sqrt{1+[f'(\bar{x}_i)]^2}\ \Delta x$$

$$SA(i^{th}\text{ cylinder}) \approx 2\pi \cdot f(\bar{x}_i) \cdot \sqrt{1+[f'(\bar{x}_i)]^2}\ \Delta x$$

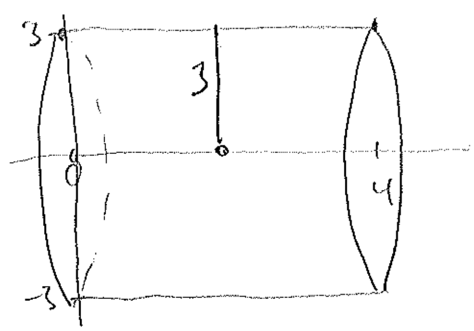
$$SA(\text{surface of revolution}) \approx \sum_{i=1}^{n} 2\pi \cdot f(\bar{x}_i)\sqrt{1+[f'(\bar{x}_i)]^2}\ \Delta x$$

$$SA = \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi f(\bar{x}_i) \cdot \sqrt{1+[f'(\bar{x}_i)]^2}\ \Delta x$$

$$SA = \int_a^b 2\pi \cdot f(x) \cdot \sqrt{1+[f'(x)]^2}\ dx$$

this is the surface area of a curve f(x) rotated about the x-axis.

ex1 find SA of curve $y=3$, $0 \le x \le 4$ rotated about x-axis

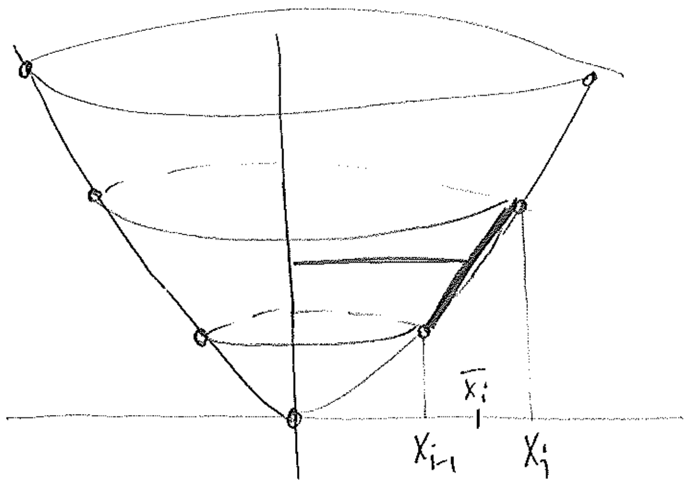


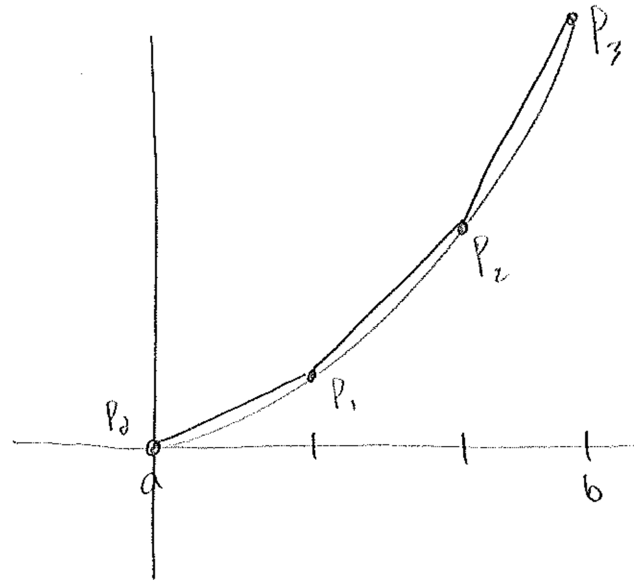
$y' = 0 \quad (y')^2 = 0$

$$SA = \int_0^4 2\pi \cdot 3\sqrt{1+0}\ dx$$

$$= \int_0^4 6\pi\ dx = 6\pi \cdot x\Big|_0^4 = 24\pi = 2\pi r \cdot h$$

Rotation of a curve about the y-axis.





$$SA(i^{th} \text{ cylinder}) \approx d(P_i, P_{i-1}) \cdot 2\pi \bar{x}_i$$

$$= 2\pi \bar{x}_i \cdot \sqrt{1 + [f'(\bar{x}_i)]^2}\ \Delta x$$

$$SA(\text{surface}) \approx \sum_{i=1}^{n} 2\pi \bar{x}_i \cdot \sqrt{1 + [f'(\bar{x}_i)]^2}\ \Delta x$$

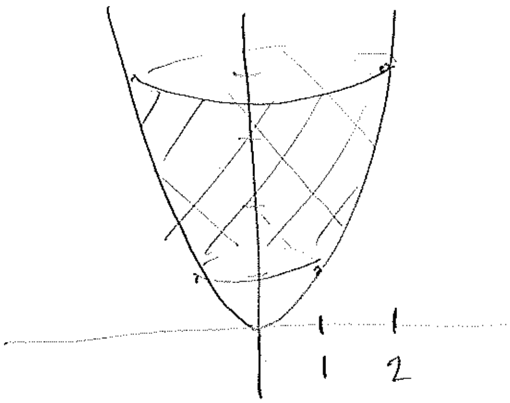
$$SA = \lim_{n \to \infty} \sum_{i=1}^{n} 2\pi \bar{x}_i \cdot \sqrt{1 + f'(\bar{x}_i)^2}\ \Delta x$$

$$= \int_a^b 2\pi x \cdot \sqrt{1 + [f'(x)]^2}\ dx$$

this is surface area of a curve f(x) rotated about the y-axis.

note: everything in terms of x.

ex) find the surface area of the curve $y = x^2$ from $(1,1)$ to $(2,4)$ rotated about the y-axis $\quad y' = 2x \quad (y')^2 = 4x^2$



$$SA = \int_1^2 2\pi x \sqrt{1+4x^2}\, dx$$

$u = 1+4x^2 \quad du = 8x\, dx \quad \frac{du}{8} = x\, dx$

$$= \frac{2\pi}{8} \int \sqrt{u}\, du = \frac{2\pi}{8} \cdot \frac{2}{3} u^{3/2}$$

$$= \frac{\pi}{6} (1+4x^2)^{3/2} \Big|_1^2 = \frac{\pi}{6}(1+16)^{3/2} - \frac{\pi}{6}(1+4)^{3/2} \approx 30.8$$