Final Review

Chp 5 : 1 2 3 4 5

Chp 6 : 1 2 3 5

Chp 7 : 1 2 3 4 8

Chp 8 : 1 2

Techniques

- sit and think
  FTC (5.3)
- u-sub (5.5)
- by parts (7.1)
- trig integrals (7.2)
- trig sub (7.3)
- partial fractions (7.4)
- improper integrals (7.8)

Applications

- area under curve (5.1)
- acc / vel / displacement vs. total distance (5.4)
- area between curves (6.1)
- Volume of solid of revolution (6.2/6.3)
  disks, washers, cylindrical shells
- avg value (6.5)
- arc length (8.1)
- Surface area of surface of revolution (8.2)
- Circles and spheres

FTC 1) If $g(x) = \int_a^x f(t)\,dt$ then $g'(x) = f(x)$ "area so far"

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

2) $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F' = f$

i.e. F is any antiderivative of f.

Note: part (2) gives us a method to evaluate definite integrals

acc/vel

displacement $= \int_a^b v(t)\,dt$

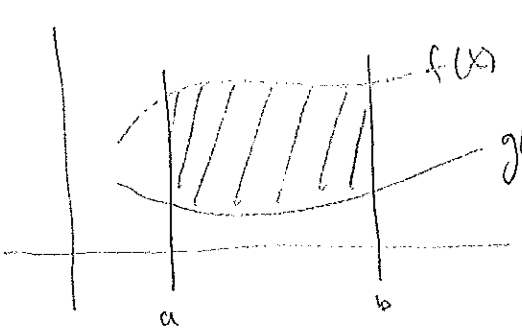
total distance $= \int_a^b |v(t)|\,dt$

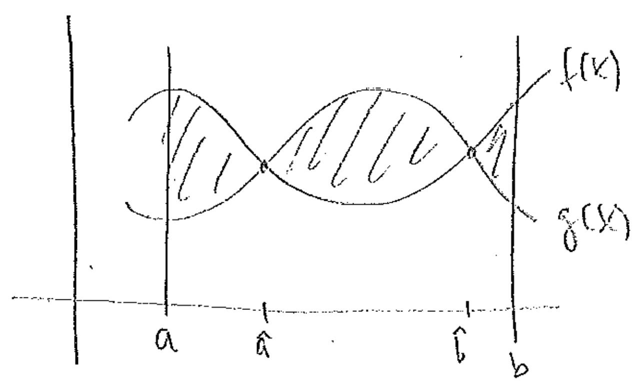
$v'(t) = a(t)$

$s'(t) = v(t)$

$s''(t) = v'(t) = a(t)$

area between curves





$$\int_a^b f(x) - g(x)\,dx$$

$$\text{Area} = \int_a^b |f(x) - g(x)|\,dx$$

$$= \int_a^{\hat{a}} f(x) - g(x)\,dx - \int_{\hat{a}}^{\hat{b}} f(x) - g(x)\,dx + \int_{\hat{b}}^{b} f(x) - g(x)\,dx$$

avg value of $f(x)$ on $[a,b]$ = $\frac{1}{b-a}\int_a^b f(x)\,dx$

by parts $\int u\,dv = uv - \int v\,du$

examples: $\int x\sin x\,dx$ $\int x^2 \ln x$ $\int \arcsin x\,dx$

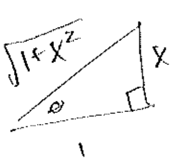
trig integrals $\int \sin^n x \cdot \cos^m x\,dx$ $\int \tan^n x \cdot \sec^m x\,dx$

method: eventually u-sub, need to rewrite via pythagorus.

trig sub

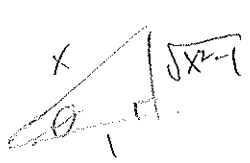
- $\int \frac{1}{\sqrt{1-x^2}}\,dx$ $x = \sin\theta$, $dx = \cos\theta\,d\theta$ $\int \frac{\cos\theta}{\cos\theta}\,d\theta = \theta + C$ $= \arcsin(x) + C$

- $\int \frac{1}{\sqrt{1+x^2}}\,dx$ $x = \tan\theta$, $dx = \sec^2\theta\,d\theta$ $\int \frac{\sec^2\theta}{\sec\theta}\,d\theta = \int \sec\theta\,d\theta = \ln(\sec\theta + \tan\theta)$



$= \ln\left(\sqrt{1+x^2} + x\right) + C$

- $\int \frac{1}{\sqrt{x^2-1}}\,dx$ $x = \sec\theta$, $dx = \sec\theta\tan\theta\,d\theta$ $\int \frac{\sec\theta\tan\theta}{\tan\theta}\,d\theta = \ln(\sec\theta + \tan\theta)$



$= \ln\left(x + \sqrt{x^2-1}\right) + C$