Arc length $= \int_a^b \sqrt{1+[f'(x)]^2}\, dx$

Rotating curve f(x) about the x-axis:

$$SA = \int_a^b 2\pi f(x) \cdot \sqrt{1+[f'(x)]^2}\, dx$$

Rotating curve f(x) about the y-axis:

$$SA = \int_a^b 2\pi x \cdot \sqrt{1+[f'(x)]^2}\, dx$$

whoa! you know how to do these integrals!

Techniques of Integration: u-sub, trig-sub, algebraic manipulation

examples: u-sub, last example on 2/4

trig-sub, circumference of circle

alg. manipulation, circumference and SA of sphere of circle ex1 find SA of curve $f(x) = \sqrt{x+1}$ $\quad 0 \le x \le 1$

rotated about x-axis.

$$y' = \frac{1}{2\sqrt{x+1}} \qquad (y')^2 = \frac{1}{4(x+1)}$$

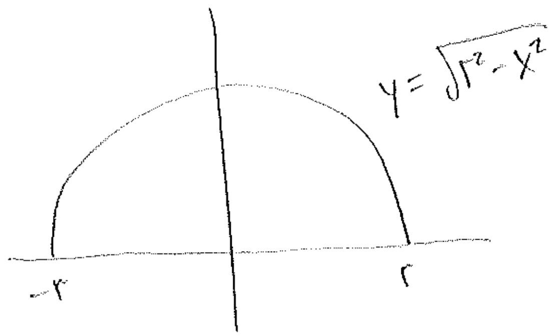
$$SA = \int_0^1 2\pi \sqrt{x+1} \cdot \sqrt{1 + \frac{1}{4(x+1)}}\, dx$$

$$= \int_0^1 2\pi \sqrt{(x+1)\cdot\left(1 + \frac{1}{4(x+1)}\right)}\, dx = \int_0^1 2\pi \sqrt{x+1+\frac{1}{4}}\, dx$$

$$= 2\pi \cdot \frac{2}{3}\left(x + \frac{5}{4}\right)^{3/2} \Big|_0^1 = 2\pi \cdot \frac{2}{3}\left(1+\frac{5}{4}\right)^{3/2} - 2\pi\cdot\frac{2}{3}\left(\frac{5}{4}\right)^{3/2} \approx 8.2$$

ex1 find surface area of a sphere with radius r

Q: does anyone know ... $4\pi r^2$



$$y' = \frac{-2x}{2\sqrt{r^2-x^2}}$$

$$(y')^2 = \frac{x^2}{r^2-x^2}$$

$$SA = \int_{-r}^{r} 2\pi\sqrt{r^2-x^2} \cdot \sqrt{1+\frac{x^2}{r^2-x^2}}\, dx$$

$$= \int_{-r}^{r} 2\pi\sqrt{r^2-x^2+x^2}\, dx = \int_{-r}^{r} 2\pi\sqrt{r^2}\, dx = \int_{-r}^{r} 2\pi r\, dx$$

$$= 2\pi r x \Big|_{-r}^{r} = 2\pi r^2 + 2\pi r^2 = 4\pi r^2$$

notes: Archimedes is first to write this down via method of exhaustion

Q: Is $4\pi r^2$ the derivative of anything?

$$\frac{d}{dr}\left[\frac{4}{3}\pi r^3\right] = 4\pi r^2 \quad \text{i.e.} \quad \int_0^r 4\pi r^2\, dr = \frac{4}{3}\pi r^3$$

think about Russian dolls --- infinitesimal sums ex1 find SA of $y = \frac{x^3}{3} + \frac{1}{4x}$ $1 \le x \le 2$ rotated about y-axis.

$y' = x^2 - \frac{1}{4x^2}$ $(y')^2 = x^4 - \frac{1}{4} - \frac{1}{4} + \frac{1}{16x^4}$

$SA = \int_1^2 2\pi x \sqrt{1 + x^4 - \frac{1}{2} + \frac{1}{16x^4}}\, dx = \int_1^2 2\pi x \sqrt{x^4 + \frac{1}{2} + \frac{1}{16x^4}}\, dx$

$\left(x^2 + \frac{1}{4x^2}\right)^2 = x^4 + \frac{1}{4} + \frac{1}{4} + \frac{1}{16x^4}$

$= \int_1^2 2\pi x \left(x^2 + \frac{1}{4x^2}\right) dx = \int_1^2 2\pi x^3 + \frac{\pi}{2x}\, dx = \frac{2\pi x^4}{4} + \frac{\pi}{2}\ln(x) \Big|_1^2$

$= \frac{2\pi 2^4}{4} + \frac{\pi}{2}\ln(2) - \frac{2\pi}{4} - 0$