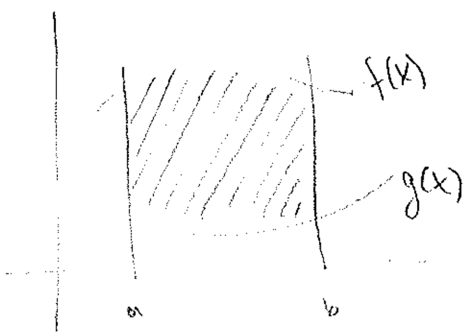
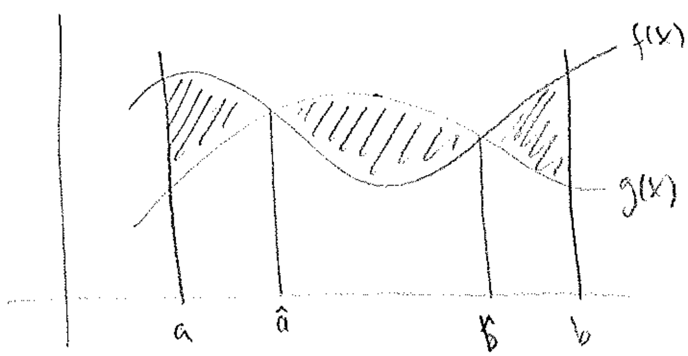
Last time: Area between curves:

FACT: the area between the curves $f(x)$ and $g(x)$ and between $x=a$ and $x=b$ is $\text{Area} = \int_a^b |f(x)-g(x)|\,dx$



when $f(x) \geq g(x)$ for $a \leq x \leq b$ then

$$\text{Area} = \int_a^b f(x)-g(x)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

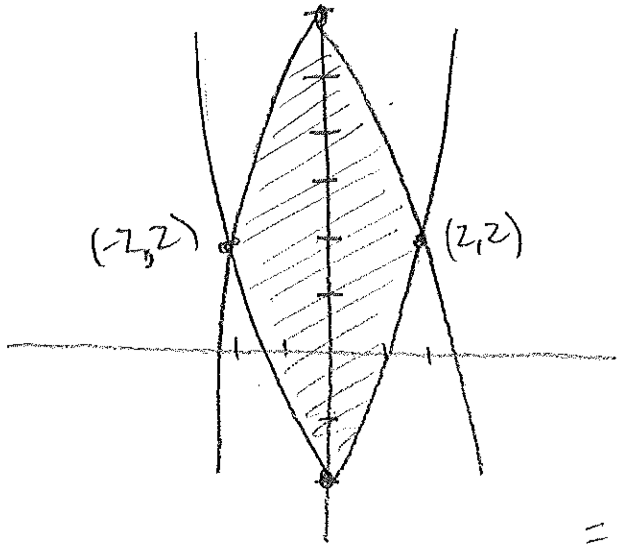


$$\text{Area} = \int_a^b |f(x)-g(x)|\,dx$$

$$= \int_a^{\hat{a}} f(x)-g(x)\,dx + \int_{\hat{a}}^{\hat{b}} g(x)-f(x)\,dx$$

$$+ \int_{\hat{b}}^{b} f(x)-g(x)\,dx$$

ex) find area enclosed by $y=6-x^2$ $y=x^2-2$



$$6-x^2=x^2-2$$
$$8=2x^2$$
$$x=\pm 2$$

$$\text{Area}=\int_{-2}^{2} 6-x^2-(x^2-2)\,dx$$

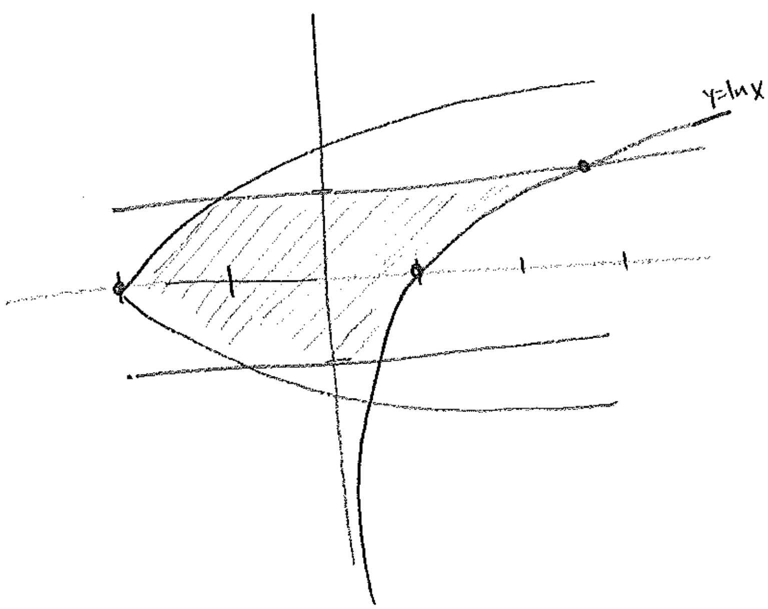
$$=\int_{-2}^{2} 8-2x^2\,dx = 8x-\tfrac{2}{3}x^3\Big|_{-2}^{2}$$

$$=16-\frac{16}{3}-\left(-16+\frac{16}{3}\right)=32-\frac{32}{3}=21\tfrac{1}{3}$$

ex) x as a function of y

find area between $x=y^2-2$ and $y=\ln x$ between $y=-1$ and $y=1$

$$x=e^y$$



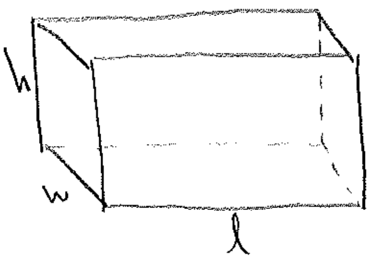
$e^y-(y^2-2)\geq 0$ on $-1\leq y\leq 1$

$$\text{Area}=\int_{-1}^{1} e^y-y^2+2\,dy$$

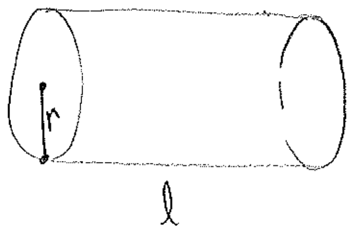
$$=e^y-\frac{y^3}{3}+2y\Big|_{-1}^{1}$$

$$=e-\frac{1}{3}+2-\left(\frac{1}{e}+\frac{1}{3}-2\right)=e-\frac{1}{e}+\frac{10}{3}$$

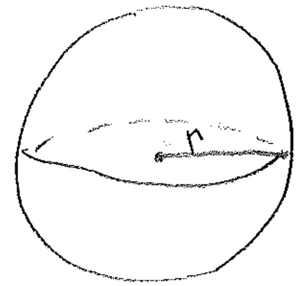
§6.2 Volumes



$V = lwh$



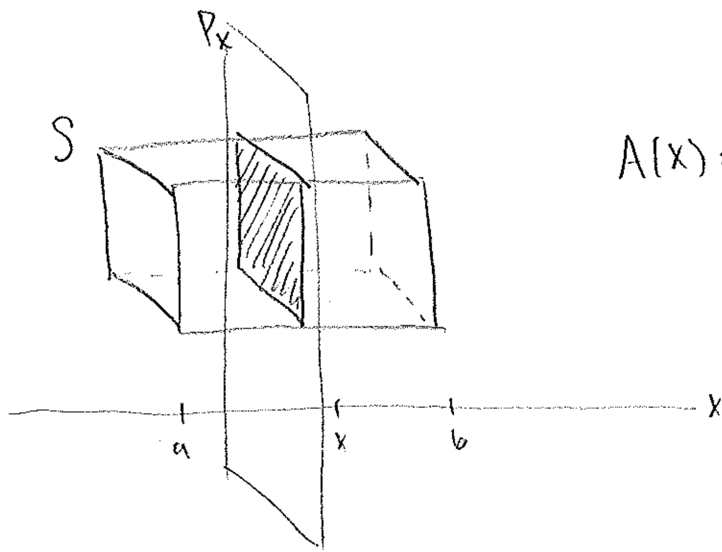
$V = \pi r^2 l$



$V = \frac{4}{3}\pi r^3$

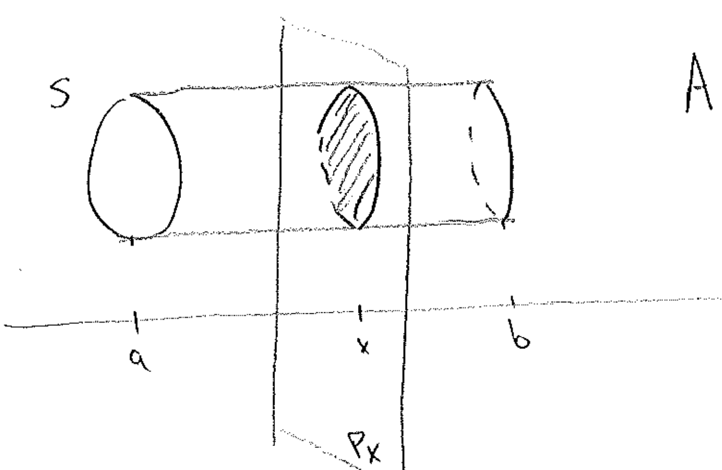
def $A(x)$ is the area of the cross-section formed by intersecting the solid S with a plane P_x which is perpendicular to the x-axis and passes through the point x, $a \le x \le b$.

ex1



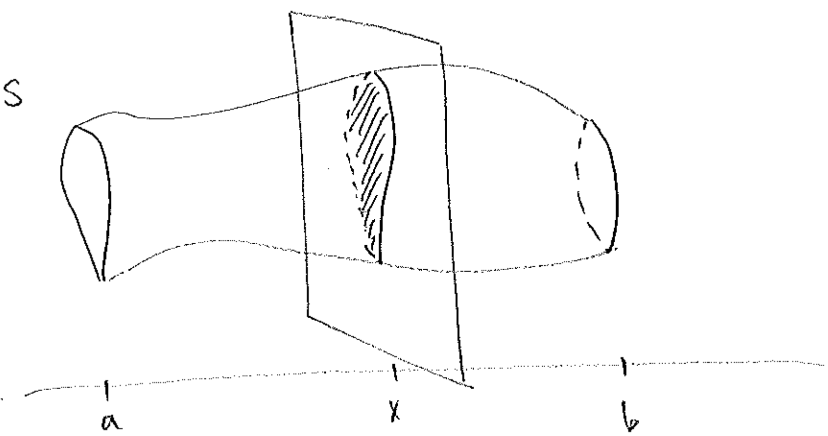
$A(x) =$ shaded region $= w \cdot h$

ex1

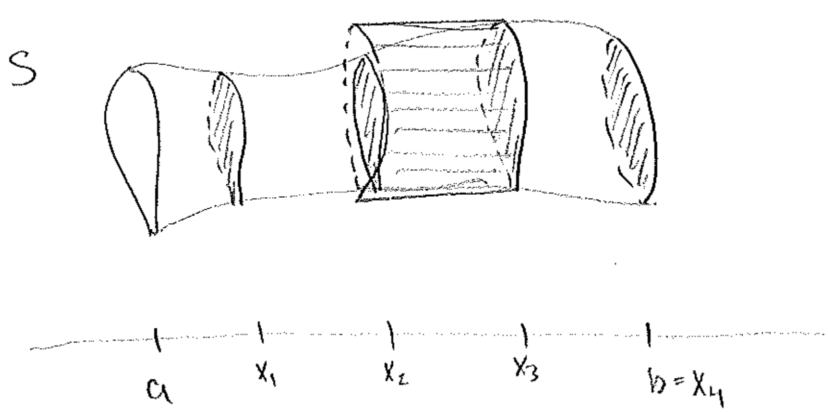


$A(x) = \pi r^2$

note: $A(x)$ will vary as x increases from a to b



idea how to use the function $A(x)$ to find Volume (S)



n - subintervals

right endpoints

- compute $A(x_i)$ for each x_i
- find $\Delta x = \frac{b-a}{n}$, then multiply
- $A(x_i) \cdot \Delta x$ = volume of i^{th} coin (slice)

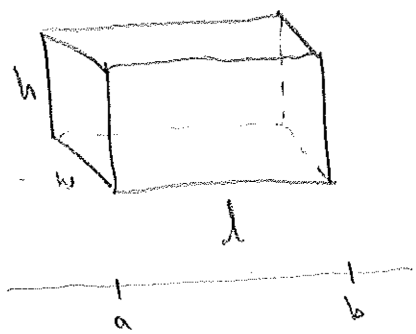
- then add volume of all slices

$$\text{Volume} \approx \sum_{i=1}^{n} A(x_i) \cdot \Delta x$$

- take limit as $n \to \infty$

$$\boxed{\text{Volume} = \lim_{n \to \infty} \sum_{i=1}^{n} A(x_i)\Delta x = \int_a^b A(x)\,dx}$$

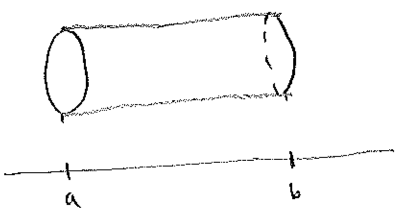
ex1



$A(x) = h \cdot w$

$$\text{Vol} = \int_a^b hw\,dx = hw(b-a) = h \cdot w \cdot l$$

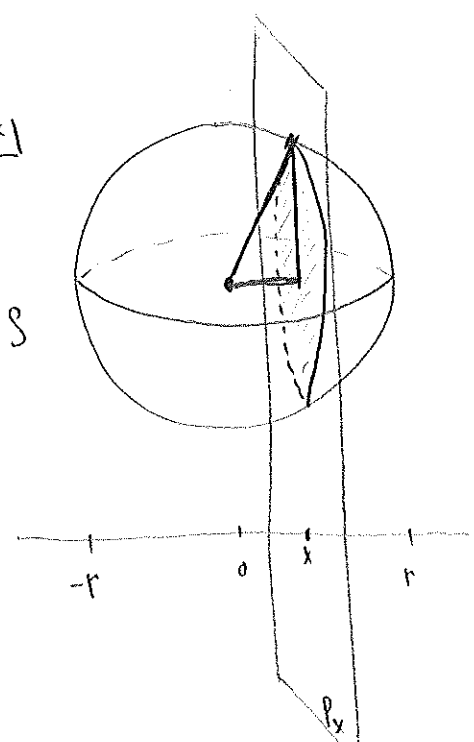
ex1

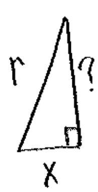


$A(x) = \pi r^2$

$$\text{Vol} = \int_a^b \pi r^2\,dx = \pi r^2 (b-a) = \pi r^2 \cdot l$$

ex1





$y = \sqrt{r^2 - x^2}$

$A(x) = \pi y^2 = \pi (r^2 - x^2)$

$$\text{Vol} = \int_{-r}^{r} A(x)\,dx = \int_{-r}^{r} \pi (r^2 - x^2)\,dx = \pi \left(xr^2 - \frac{x^3}{3} \right) \Bigg|_{-r}^{r}$$

$$= \pi \left(r^3 - \tfrac{1}{3} r^3 \right) - \pi \left(-r^3 + \tfrac{1}{3} r^3 \right) = \frac{4}{3} \pi r^3$$