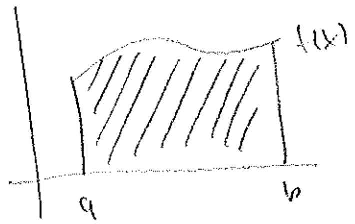
## § 6.1 Area Between Curves

Review:



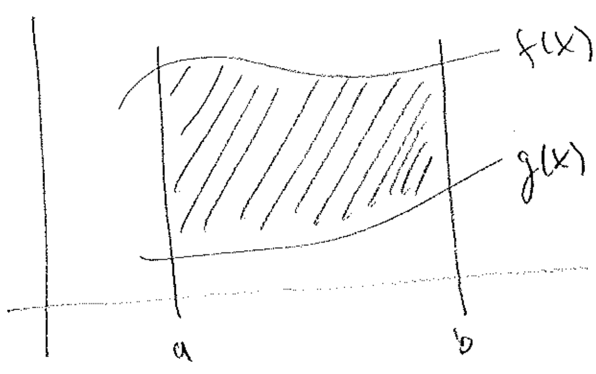

then Area $= \int_a^b f(x)\,dx$

note: actual equality, not just an estimate via Riemann sums.

FTC, 2 $\rightarrow$ integrals are actually easy to compute, only need to find antiderivative of $f(x)$.

Substitution: first of many methods we will learn that allow us to integrate more complicated functions.

New Question: Area between curves

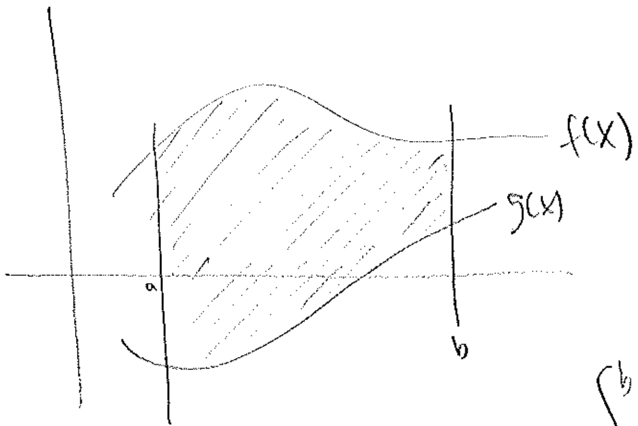



$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b f(x) - g(x)\,dx$$

Fact: the area between the curves $f(x)$ and $g(x)$ and between the lines $x = a$ and $x = b$, where $f(x) \geq g(x)$ for $x \in [a,b]$ is

$$\text{Area} = \int_a^b f(x) - g(x)\,dx$$

note: this formula holds even if the functions are NOT positive

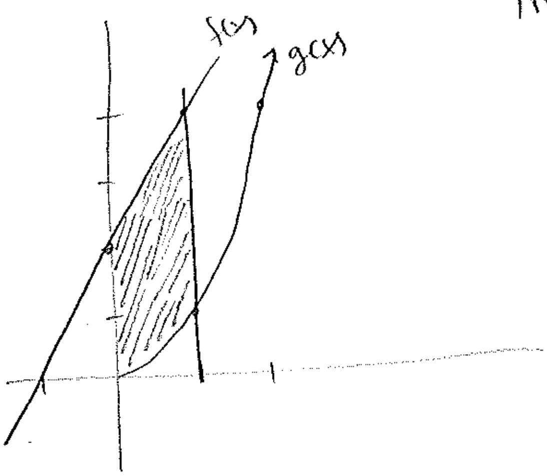



$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b f(x)-g(x)\,dx$$

think about interpretations of area

$\int_a^b g(x)\,dx =$ area Red − area Purple

ex| find area between $f(x) = 2x+2$ and $g(x) = x^2$ between $x=0$ and $x=1$

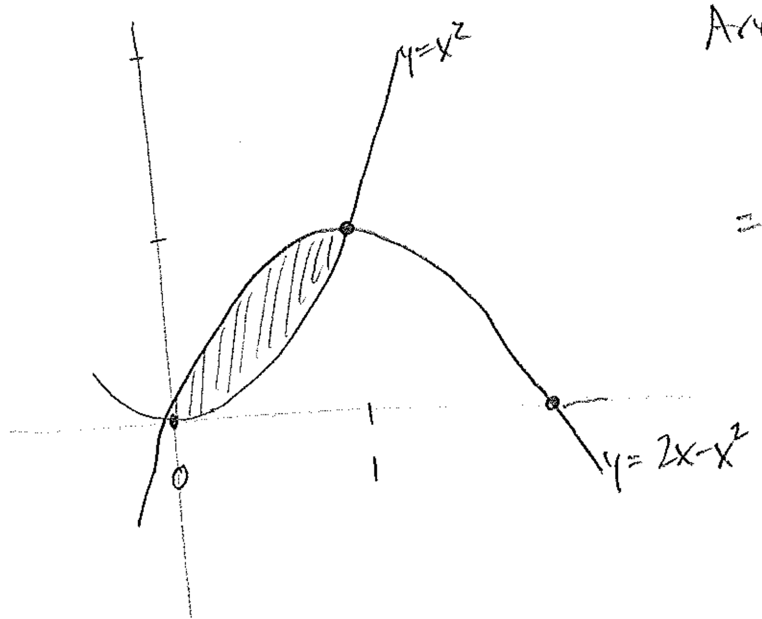



$$\text{Area} = \int_0^1 2x+2-x^2\,dx$$

$$= x^2+2x-\frac{x^3}{3}\Big|_0^1 = 1+2-\frac{1}{3} = 2\tfrac{2}{3}$$

ex) find area enclosed by the parabolas

$y = x^2$ and $y = 2x - x^2$



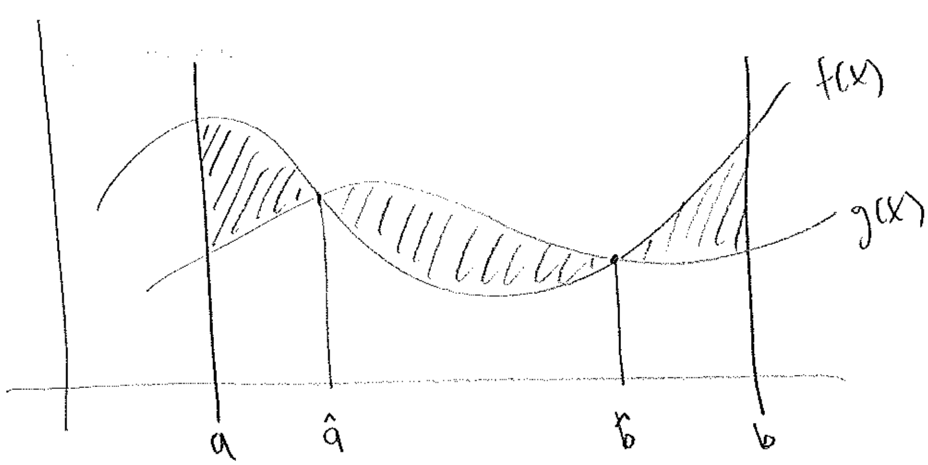

$$\text{Area} = \int_0^1 2x - x^2 - x^2 \, dx$$

$$= \int_0^1 2x - 2x^2 \, dx = x^2 - \frac{2}{3}x^3 \Big|_0^1$$

$$= 1 - \frac{2}{3} - (0) = 1/3$$

Question: What happens when $f(x) \geq g(x)$ for some $x$

but $g(x) \geq f(x)$ for other values of $x$?



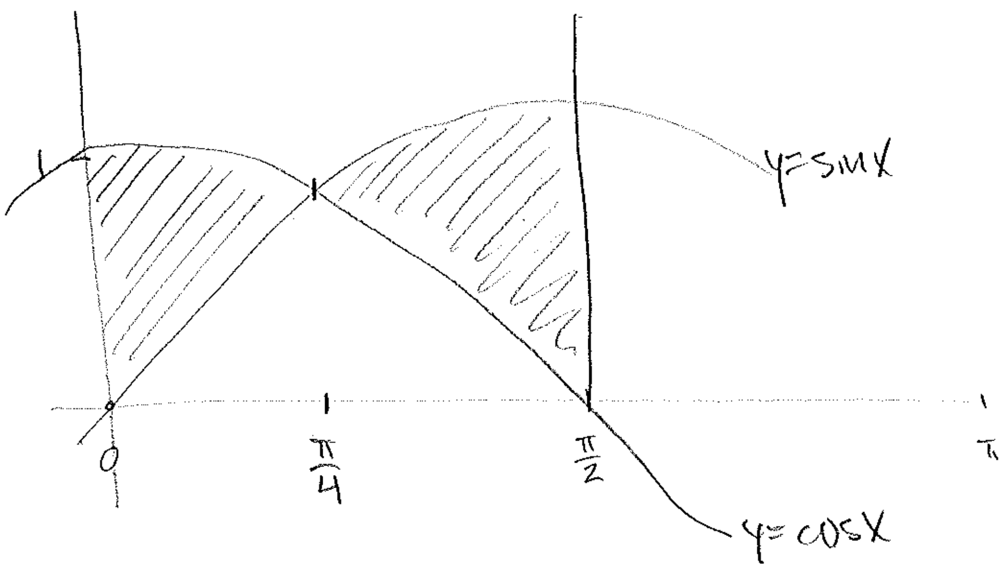

Area "enclosed" "between" "bounded" by the two

curves $f(x)$ and $g(x)$ between $x = a$ and $x = b$

Fact: the area between the curves $y=f(x)$ and $y=g(x)$ between $x=a$ and $x=b$ is $\text{Area} = \int_a^b |f(x)-g(x)|\,dx$

note: we no longer need $f(x) \geq g(x)$, nor do we require positive functions.

ex] Find the area bounded by the curves $y=\sin x$ and $y=\cos x$ between $x=0$ and $x=\pi/2$




$$\text{Area} = \int_0^{\pi/2} |\cos x - \sin x|\,dx = \int_0^{\pi/4} \cos x - \sin x\,dx + \int_{\pi/4}^{\pi/2} \sin x - \cos x\,dx$$

$$= \sin x + \cos x \Big|_0^{\pi/4} + -\cos x - \sin x \Big|_{\pi/4}^{\pi/2}$$

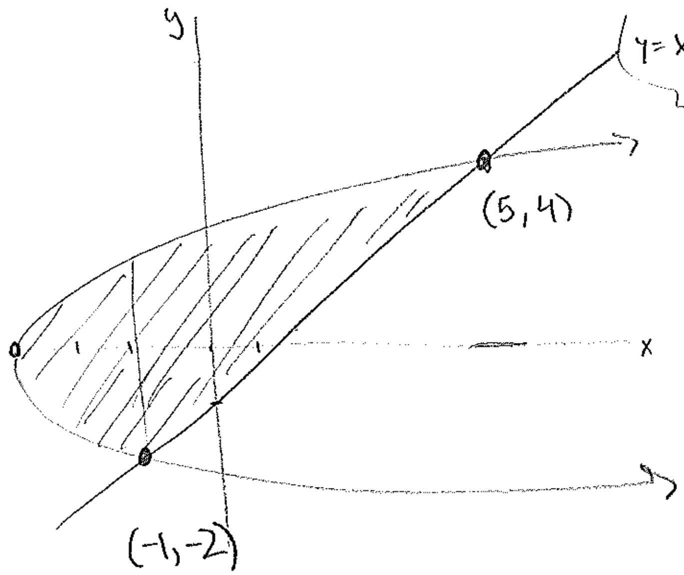
$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0+1) + \left[0 - 1 - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\right] = 2\sqrt{2} - 2$$

## Area between curves

x as a function of y

ex. Find area enclosed by line $y = x-1$ and parabola $y^2 = 2x+6$

$$x = \frac{y^2-6}{2}$$




$x = y+1 \quad f(y) = y+1$

$x = \frac{y^2-6}{2} \quad g(y) = \frac{1}{2}y^2 - 3$

intersection points?

$(x-1)^2 = 2x+6$

$x^2 - 2x + 1 - 2x - 6 = 0$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$$\text{Area} = \int_{-2}^{4} |f(y) - g(y)|\, dy = \int_{-2}^{4} f(y) - g(y)\, dy$$

$$= \int_{-2}^{4} y+1 - \left(\tfrac{1}{2}y^2 - 3\right) dy = \int_{-2}^{4} -\tfrac{1}{2}y^2 + y + 4\, dy$$

$$= -\tfrac{1}{6}y^3 + \tfrac{1}{2}y^2 + 4y \Big|_{-2}^{4} = -\tfrac{1}{6}(64) + \tfrac{1}{2}(16) + 16 - \left(\tfrac{-1}{6}(8) + 2 - 8\right) = 18$$