lecture notes 1/23

last time: Substitution

$$\int \frac{x^2}{x^3+1}\,dx \qquad u = x^3+1 \qquad du = 3x^2\,dx \qquad \tfrac{1}{3}\,du = x^2 dx$$

$$= \int \frac{\frac{1}{3}\,du}{u} = \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3+1| + C$$

Check: $\frac{d}{dx}\left\{ \frac{1}{3}\ln|x^3+1| + C \right\} = \frac{1}{3}\cdot\frac{3x^2}{x^3+1} = \frac{x^2}{x^3+1}$

Substitution is Chain Rule in reverse.

Substitution and Definite Integrals

ex1 $\displaystyle\int_1^2 \frac{e^{1/x}}{x^2}\,dx \qquad u = \frac{1}{x} \qquad du = -x^{-2}\,dx \qquad -du = \frac{1}{x^2}\,dx$

$$= -\int_{x=1}^{x=2} e^u\,du = -e^u\Big|_{x=1}^{x=2} = -e^{1/x}\Big|_1^2 = -e^{1/2} + e^1$$

idea: $\displaystyle -\int_1^2 e^u\,du = -e^u\Big|_1^2 = -e^2 + e^1$ WRONG note: Stewart has another method for definite integral via substitution... we omit this method.

ex] $\int_1^e \frac{\ln x}{x}\,dx$ $\quad u = \ln x \quad du = \frac{1}{x}\,dx$

$$= \int_{x=1}^{x=e} u\,du = \frac{1}{2}u^2 \Big|_{x=1}^{x=e} = \frac{1}{2}(\ln x)^2 \Big|_1^e = \frac{1}{2}(\ln e)^2 - \frac{1}{2}(\ln 1)^2 = \frac{1}{2}$$

ex] $\int \sqrt{2x+1}\,dx$ $\quad u = 2x+1 \quad du = 2\,dx \quad \frac{1}{2}du = dx$

$$= \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = \frac{1}{3}(2x+1)^{3/2} + C$$

note: Chain Rule in reverse

ex] $\int_0^{\pi/2} \cos^4 x \cdot \sin x\,dx$ $\quad u = \cos x \quad du = -\sin x\,dx \quad -du = \sin x\,dx$

$$= -\int_{x=0}^{x=\pi/2} u^4\,du = -\frac{1}{5}u^5 \Big|_{x=0}^{x=\pi/2} = -\frac{1}{5}\cos^5 x \Big|_0^{\pi/2} = 0 - \left(-\frac{1}{5}\cdot 1\right) = \frac{1}{5}$$

Concludes Chp 5 and material for Midterm # 1.

Midterm 1 Review

Section 5.1 - Areas and Distances

1. (a) (Stewart, 5.1: 5) Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right endpoints. Then improve your estimate by using six rectangles. Sketch the curve and the approximating rectangles.

   (b) Repeat part (a) using left endpoints.

   (c) Repeat part (a) using midpoints.

   (d) From your sketches in parts (a)-(c), which appears to be the best estimate?

2. (Stewart, 5.1: 13) The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these three seconds.

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

3. (Stewart, 5.1: 19) Find an expression for the area under the graph of f as a limit. Do not evaluate the limit.

   (a) $f(x) = \frac{2x}{x^2+1}$, $1 \leq x \leq 3$

   (b) $f(x) = \sqrt{\sin x}$, $0 \leq x \leq \pi$

Section 5.2 - The Definite Integral

1. (Stewart, 5.2: 26)

   (a) Find an approximation to the integral $\int_0^4 (x^2 - 3x)dx$ using a Riemann sum with right endpoints and $n = 8$.

   (b) Draw a diagram showing the approximating rectangles in part (a).

   (c) Interpret the integral in part (a) as a difference of areas and illustrate with a diagram.

2. (Stewart, 5.2: 35, 37, 39) Evaluate the integral by interpreting it in terms of areas.

   (a) $\int_{-1}^{2} (1 - x)dx$

   (b) $\int_{-3}^{0} (1 + \sqrt{9 - x^2})dx$

   (c) $\int_{-1}^{2} |x| dx$

3. (Stewart, 5.2: 47) Write as a single integral in the form $\int_a^b f(x)dx$:

$$\int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx$$

4. Given that $\int_0^{2\pi} f(x)dx = 4$ find the value for the following integrals:

   (a) $\int_0^{2\pi}(3f(x)+2)\,dx$
   (b) $\int_0^{2\pi}(\frac{2f(x)}{3}+5)\,dx$

5.3 - The Fundamental Theorem of Calculus

1. What does the Fundamental Theorem of Calculus say? (Both parts)

2. (Stewart, 5.3: 7,9,11,12,13) Find the derivative of the given functions.

   (a) $g(x) = \int_1^x \frac{1}{t^3+1}\,dt$
   (b) $g(s) = \int_5^s (t-t^2)^8\,dt$
   (c) $F(x) = \int_x^\pi \sqrt{1+\sec t}\,dt$
   (d) $G(x) = \int_x^1 \cos\sqrt{t}\,dt$

3. (Stewart, 5.3: 75) On what interval is the curve

$$y = \int_0^x \frac{t^2}{t^2+t+2}\,dt$$

   concave downward?

5.4 - Indefinite Integrals

1. (Stewart, 5.4: 7, 9, 15) Find the general indefinite integral.

   (a) $\int(x^4 - \frac{1}{2}x^3 + \frac{1}{4}x - 2)\,dx$
   (b) $\int(u+4)(2u+1)\,du$
   (c) $\int(\theta - \csc\theta\cot\theta)\,d\theta$

2. (Stewart, 5.3: 35, 37, 43) Evaluate the integral

   (a) $\int_1^2 \frac{v^3+3v^6}{v^4}\,dv$
   (b) $\int_0^1 (x^e + e^x)\,dx$
   (c) $\int_0^\pi f(x)\,dx$ where $f(x) = \sin x$ if $0 \le x < \pi/2$ and $f(x) = \cos x$ if $\pi/2 \le x \le \pi$

3. (Stewart, 5.4: 21,25,33,35,38) Evaluate the integral.

   (a) $\int_{-2}^3 (x^2-3)\,dx$
   (b) $\int_0^2 (2x-3)(4x^2+1)\,dx$
   (c) $\int_1^2 (\frac{x}{2} - \frac{2}{x})\,dx$
   (d) $\int_0^1 (x^{10} + 10^x)\,dx$
   (e) $\int_0^{\pi/3} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta$

4. (Stewart, 5.4: 59) The velocity function (in meters per second) for a particle moving along a line is $v(t) = 3t^2 + 2t - 5$ for $0 \leq t \leq 3$. Find:

   (a) The displacement during the given time interval.

   (b) The distance traveled by the particle during the given time interval.

5. (Stewart, 5.4: 61) The acceleration function in (m/s^2) for a particle moving along a line is $a(t) = t + 4$, the initial velocity is $v(0) = 5$, and the initial position is $s(0) = 3$.

   (a) Find the velocity at time t

   (b) Find the position at time t

   (c) Find the distance traveled during the given time interval

5.5 - The Substitution Rule

1. Decide whether or not you need to use substitution for the following integrals and then evaluate them appropriately.

   (a) $\int e^{7x}\, dx$

   (b) $\int (8x^3 + 3x^2)\, dx$

   (c) $\int \cos(x/2)\, dx$

   (d) $\int e^{x^2} x\, dx$

   (e) $\int y(y^2 + 1)^2\, dy$

   (f) $\int_0^1 x\sqrt{1 - x^2}\, dx$

   (g) $\int_1^e \frac{\ln x}{x}\, dx$

   (h) $\int_1^3 (\frac{1-x}{x})^2\, dx$

   (i) $\int_{-1}^2 \frac{x}{1+x^2}\, dx$

   (j) $\int_0^{\pi/6} \cos^3(2x)\sin(2x)\, dx$

   (k) $\int_{-1}^2 (x + 2)^2\, dx$

   (l) $\int_0^2 \frac{e^x}{1+e^x}\, dx$

   (m) $\int_0^{\pi} \frac{\sec(3x)\tan(3x)}{\cos(3x)}\, dx$

   (n) $\int_3^4 (3 - x)^{10}\, dx$