Schedule

Week 3

1/21 No Class. WW 5.3 due

1/22 x-hour

1/23 Quiz 2 (5.3 + 5.4) WW 5.4 due

1/25 HW 2 due

Week 4

1/28 WW 5.5 due

1/29 x-hour Review Midterm 7-9pm Wilder 111

1/30 NO Quiz

2/1 HW3 due WW 6.1 due

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So we can regard an indefinite integral as representing an entire *family* of functions (one antiderivative for each value of the constant C).

You should distinguish carefully between definite and indefinite integrals. A definite integral $\int_a^b f(x)\,dx$ is a *number*, whereas an indefinite integral $\int f(x)\,dx$ is a *function* (or family of functions). The connection between them is given by Part 2 of the Fundamental Theorem: If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = \int f(x)\,dx \Big]_a^b$$

The effectiveness of the Fundamental Theorem depends on having a supply of antiderivatives of functions. We therefore restate the Table of Antidifferentiation Formulas from Section 4.9, together with a few others, in the notation of indefinite integrals. Any formula can be verified by differentiating the function on the right side and obtaining the integrand. For instance,

$$\int \sec^2 x\,dx = \tan x + C \qquad \text{because} \qquad \frac{d}{dx}(\tan x + C) = \sec^2 x$$

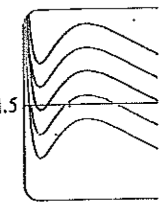

FIGURE 1

The indefinite integral in
in Figure 1 for several va
value of C is the y-inter

1 Table of Indefinite Integrals

$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\cancel{\int \sinh x\,dx = \cosh x + C} \qquad \cancel{\int \cosh x\,dx = \sinh x + C}$$

Recall from Theorem 4.9.1 that the most general antiderivative *on a given interval* is obtained by adding a constant to a particular antiderivative. **We adopt the convention that when a formula for a general indefinite integral is given, it is valid only on an interval.** Thus we write

$$\int \frac{1}{x^2}\,dx = -\frac{1}{x} + C$$

Differentiation and Integration as Inverse Processes

Fund Thm Calc: Suppose $f(x)$ continuous on $[a,b]$.

1. If $g(x) = \int_a^x f(t)\,dt$ then $g'(x) = f(x)$.

2. $\int_a^b f(x)\,dx = F(b) - F(a)$ where F any antiderivative of f
   i.e., $F' = f$.

Notes: 1 says $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

2 says, since $F'(x) = f(x)$, that $\int_a^b F'(x)\,dx = F(b) - F(a)$

Indefinite Integral: Section 5.4

FTC shows how important the antiderivative is. We need a notation for antiderivative of $f(x)$. We use $\int f(x)\,dx$ and call this the indefinite integral.

Thus $\int f(x)\,dx = F(x)$ means $F'(x) = f(x)$.

ex1 $\int x^2\,dx = \frac{x^3}{3} + C$

Note: the indefinite integral represents a family of functions, one antiderivative for each value of the constant C.

Definite vs. Indefinite Integral

Definite Integral is a number : $\int_0^1 x^2\,dx = \frac{1}{3}$

Indefinite Integral is a function (or family of functions) : $\int x^2\,dx = \frac{x^3}{3} + C$

The connection is given by FTC, Part 2 $\int_a^b f(x)\,dx = F(b) - F(a)$

where F is any antiderivative, i.e. $\int_a^b f(x)\,dx = \int f(x)\,dx \Big|_a^b$

When do we need $+C$?

You will never have a number $+C$, i.e. $\frac{1}{3} + C$

Always a function $+C$ i.e. $\frac{x^3}{3} + C$

What about when applying FTC, Part 2

$$\int_0^1 x^2\,dx = \int f(x)\,dx \Big|_0^1 = \frac{x^3}{3} + C \Big|_0^1 = \frac{1}{3} + C - (0 + C) = \frac{1}{3}$$

ex) Find the indefinite integral : $\int (10x^4 - 2\sec^2 x)\,dx$

A: $\int (10x^4 - 2\sec^2 x)\,dx = 10\int x^4\,dx - 2\int \sec^2 x\,dx = 10\frac{x^5}{5} - 2\tan x + C$

ex) Evaluate $\int_0^3 (x^3 - 6x)\,dx$

A: $\int_0^3 x^3 - 6x\,dx = \frac{x^4}{4} - 6\frac{x^2}{2}\Big|_0^3 = \left(\frac{3^4}{4} - \frac{6\cdot 3^2}{2}\right) - (0) = -6.75$

The distance/velocity problem (see 5.4.60 and 62)

A particle moves along a line with velocity $v(t) = t^2 - t - 6$

a) find displacement of the particle during period $1 \le t \le 4$

b) find total distance traveled during this period.

A: displacement $= s(4) - s(1) = \int_1^4 v(t)\,dt = \int_1^4 t^2 - t - 6$

$= \frac{t^3}{3} - \frac{t^2}{2} - 6t \Big|_1^4 = -\frac{9}{2}$

note that velocity is both positive and negative on this interval.

$v(t) = t^2 - t - 6 = (t-3)(t+2)$ hence $v(t) = 0$ for $t = \{-2, 3\}$

Also $v(1) = -6$ $v(4) = 6$ hence $v(t)$ positive on $(3, 4]$ and negative on $[1, 3)$.

Thus distance traveled is given by $\int_1^4 |v(t)|\,dt =$

$\int_1^3 -v(t)\,dt + \int_3^4 v(t)\,dt = -\frac{t^3}{3} + \frac{t^2}{2} + 6t \Big|_1^3 + \frac{t^3}{3} - \frac{t^2}{2} - 6t \Big|_3^4$

$= [-9 + \frac{9}{2} + 18] - [\frac{-1}{3} + \frac{1}{2} + 6] + [\frac{64}{3} - \frac{16}{2} - 24] - [9 - \frac{9}{2} - 18]$

$= \frac{22}{3} + \frac{17}{6} = \frac{61}{6}$

note: $\frac{17}{6} - \frac{22}{3} = -\frac{9}{2}$

The distance / velocity / acceleration problem

Initial velocity and acceleration are given for a particle moving in a line.

$a(t) = t+4$, $v(0) = 5$ $\quad 0 \le t \le 10$

a) find velocity at time t

b) find the displacement and total distance traveled.

A: the rate of change of velocity is the acceleration, i.e. $v'(t) = a(t)$

hence $v(t)$ is antiderivative of $a(t)$.

$$v(t) = \int a(t)\,dt = \int t+4\,dt = \tfrac{1}{2}t^2 + 4t + C$$

know $v(0) = 5 \Rightarrow 5 = \tfrac{1}{2}0^2 + 4(0) + C \Rightarrow 5 = C$

Thus $v(t) = \frac{1}{2}t^2 + 4t + 5$

$$\text{displacement} = s(10) - s(0) = \int_0^{10} \tfrac{1}{2}t^2 + 4t + 5\,dt = \left.\frac{t^3}{6} + 2t^2 + 5t\right|_0^{10}$$

$$= \frac{1000}{6} + 200 + 50 - (0) = 416\tfrac{2}{3}$$

note: distance = displacement in this example b/c velocity is positive on the interval $0 \le t \le 10$, i.e. $v(t) = |v(t)|$

so $\text{displacement} = \int_0^{10} v(t)\,dt = \int_0^{10} |v(t)|\,dt = \text{distance}$

ex) Evaluate $\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2}\, dt$

A: $\int_1^9 \frac{2t^2+t^2\sqrt{t}-1}{t^2}\, dt = \int_1^9 2 + t^{1/2} - t^{-2}\, dt = 2t + \frac{2}{3}t^{3/2} + t^{-1} \Big|_1^9 = 32\,4/9$

ex) $\int_{-1}^{1} t(1-t)^2\, dt = \int_{-1}^{1} t(1-2t+t^2)\, dt = \int_{-1}^{1} t - 2t^2 + t^3\, dt$

$= \frac{1}{2}t^2 - \frac{2}{3}t^3 + \frac{1}{4}t^4 \Big|_{-1}^{1} = \cancel{\frac{1}{2}} - \frac{2}{3} + \cancel{\frac{1}{4}} - \left(\cancel{\frac{1}{2}} + \frac{2}{3} + \cancel{\frac{1}{4}}\right) = -\frac{4}{3}$

ex) $\int x + \frac{1}{x}\, dx = \frac{x^2}{2} + \ln|x| + C$

ex) $\int \frac{\cos\theta}{\sin^2\theta}\, d\theta = \int \frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta}\, d\theta = \int \csc\theta \cdot \cot\theta\, d\theta = -\csc\theta + C$