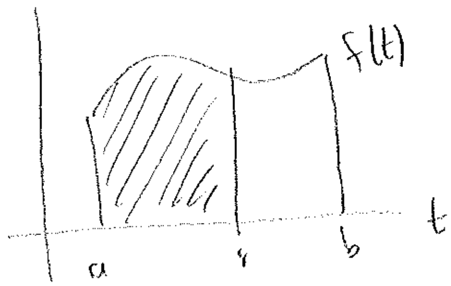
Lecture Notes 1/16

Last time: "Area so far" $g(x) = \int_a^x f(t)\,dt$ for $a \le x \le b$



Main question: what is $g'(x)$

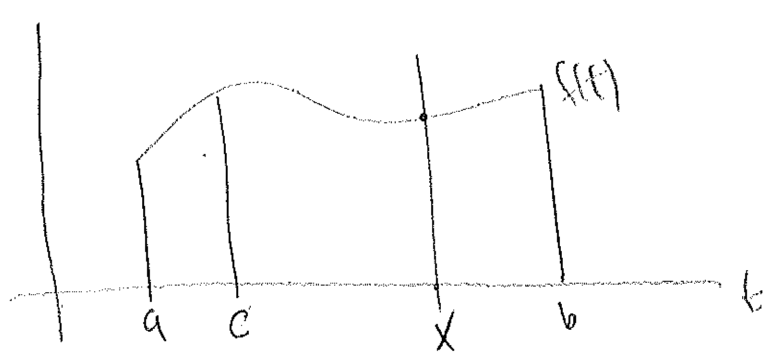
<u>Thm</u> (FTC, Part 1): If $g(x) = \int_a^x f(t)\,dt$ for $a \le x \le b$ then

$$g'(x) = f(x).$$

<u>ex1</u> If $g(x) = \int_3^x \frac{1}{t^4+1}\,dt$ find $g'(x)$

A: $g'(x) = \frac{1}{x^4+1}$

note: the lower limit of integration is important when computing $g(x)$ but NOT $g'(x)$.



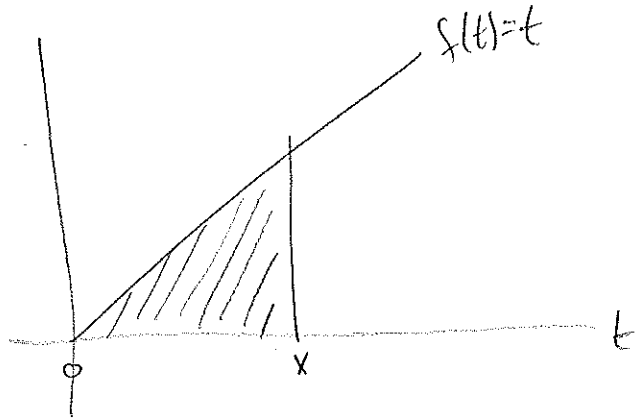
then $\int_a^x f(t)\,dt \neq \int_c^x f(t)\,dt$

but $\frac{d}{dx}\int_a^x f(t)\,dt = \frac{d}{dx}\int_c^x f(t)\,dt = f(x)$

ex) note: Revision: NOT on quiz/exam, still on HW 5.3.16

$\frac{d}{dx}\int_1^{x^4} \sec t\,dt = \frac{d}{du}\left[\int_1^u \sec t\,dt\right]\frac{du}{dx} = \sec x^4 \cdot 4x^3$

ex) $g(x) = \int_0^x t\,dt$



expression for $g(x)$

$g(x) = \frac{1}{2}x^2$ so that $g'(x) = x = f(x)$

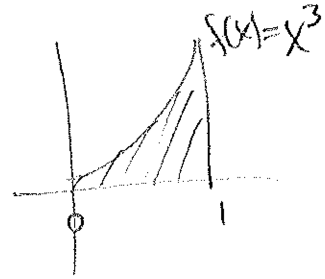
Confusion: x vs. t

$\int_3^7 f(x)\,dx = \int_3^7 f(t)\,dt$

<u>Thm</u> (FTC, 2): If f continuous on $[a,b]$ then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$ where F is <u>any</u> antiderivative of f, i.e. $F' = f$.

<u>ex</u> $\int_0^1 x^2\,dx$. Here $f(x) = x^2$ and $F(x) = \frac{x^3}{3}$ is an antiderivative



Thus $\int_0^1 x^2\,dx = F(1) - F(0) = \frac{1}{3}$

note: $G(x) = \frac{x^3}{3} + 2$ is also an antiderivative of $f(x)$.

so $$\int_0^1 x^2\,dx = G(1) - G(0) = \left(\tfrac{1}{3} + 2\right) - (0+2) = \tfrac{1}{3}$$

Notation: we will write $F(x)\Big|_a^b = F(b) - F(a)$

so that $$\int_a^b f(x)\,dx = F(x)\Big|_a^b$$ where $F'(x) = f(x)$

ex1 $\int_1^3 (x+2)(x-3)\,dx = \int_1^3 x^2 - x - 6\,dx = \left.\frac{x^3}{3} - \frac{x^2}{2} - 6x\right|_1^3$

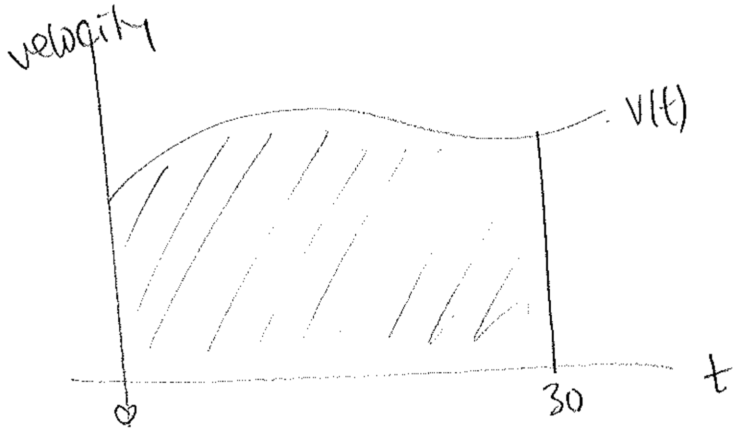
$= 9 - \frac{9}{2} - 18 - \left(\frac{1}{3} - \frac{1}{2} - 6\right) = \frac{-22}{3}$

Physical Interpretation: let $v(t)$ velocity and $s(t)$ position functions of a particle moving in a straight line.

Then $v(t) = s'(t)$, i.e. rate of change of position is velocity.

Hence $s(t)$ is antiderivative of $v(t)$.

Recall the distance problem



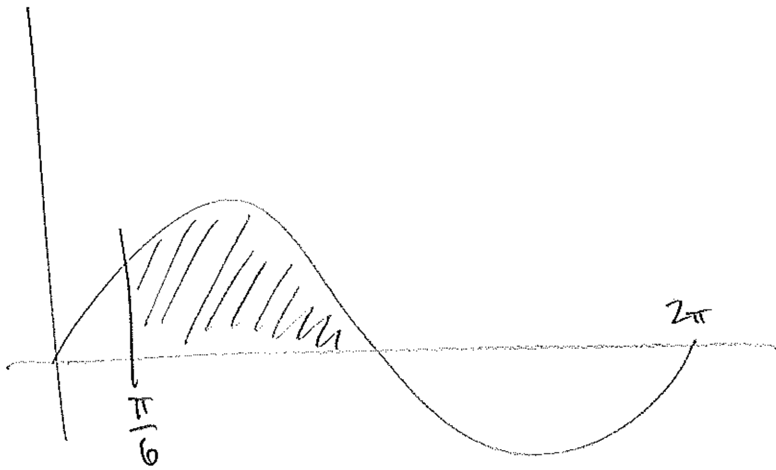
Area ⇝ distance traveled in 30s interval

(assuming positive velocity).

Then $\int_0^{30} v(t)\,dt = s(30) - s(0)$

note: we will return to this distance/velocity problem on Friday.

ex) $\int_{\pi/6}^{\pi} \sin\theta \, d\theta = -\cos\theta \Big|_{\pi/6}^{\pi} = 1 - \left(\frac{-\sqrt{3}}{2}\right) = 1 + \frac{\sqrt{3}}{2}$



ex) $\int_{0}^{2\pi} \sin\theta \, d\theta = -\cos\theta \Big|_{0}^{2\pi} = -1 - (-1) = 0$