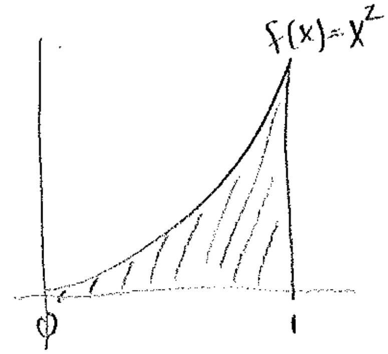
Lecture Notes 1/9

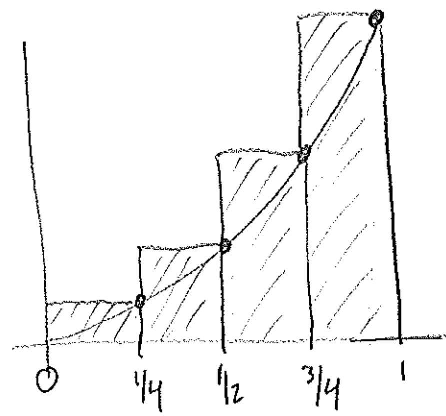
Area: History: Greek method of exhaustion

Eudoxus and Archimedes



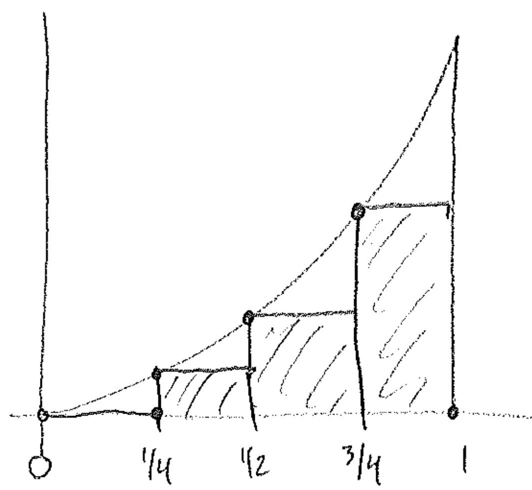
Area under parabola $f(x)=x^2$ from 0 to 1

Right endpoint approx.



Area $\approx \frac{1}{4}\left(\frac{1}{4}\right)^2 + \frac{1}{4}\left(\frac{1}{2}\right)^2 + \frac{1}{4}\left(\frac{3}{4}\right)^2 + \frac{1}{4}(1)^2 \approx .47$ (overestimate)

Left endpoint approx.

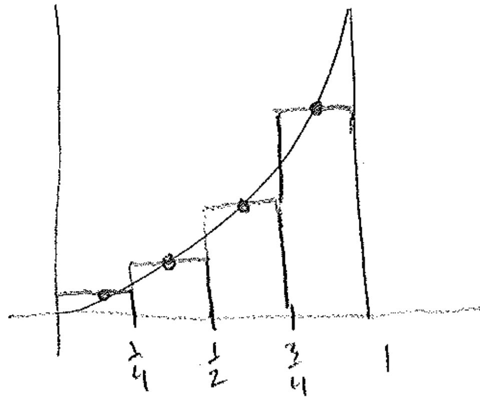


Area $\approx \frac{1}{4}(0)^2 + \frac{1}{4}\left(\frac{1}{4}\right)^2 + \frac{1}{4}\left(\frac{1}{2}\right)^2 + \frac{1}{4}\left(\frac{3}{4}\right)^2 \approx .22$

(underestimate).

Q: How can we produce a better estimate?

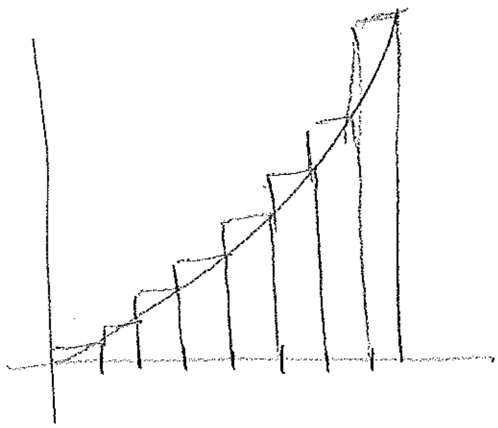
Midpoints approx:

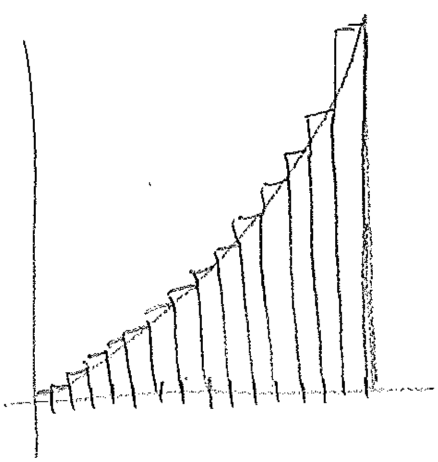


$$\text{Area} \approx \frac{1}{4}\left(\frac{1}{8}\right)^2 + \frac{1}{4}\left(\frac{3}{8}\right)^2 + \frac{1}{4}\left(\frac{5}{8}\right)^2 + \frac{1}{4}\left(\frac{7}{8}\right)^2 \approx .33$$

over or under estimate? $\Rightarrow$ ambiguous

<u>More rectangles</u>:

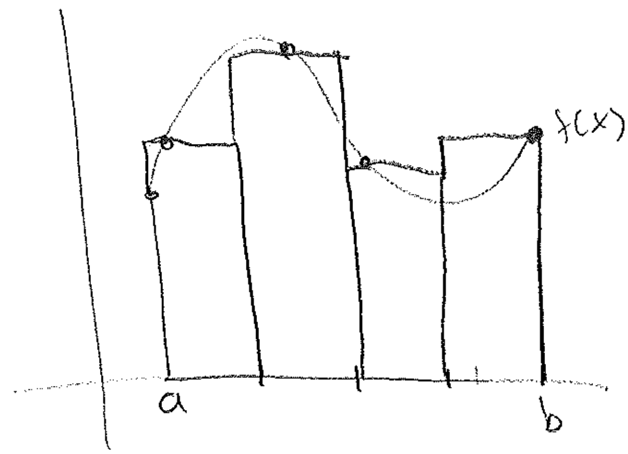




Using right endpts for this <u>increasing</u> function we still get overestimates, but the estimates are getting closer to the true value of the area.

<u>Definition</u>: the <u>Area</u> of the region that lies under the graph of a continuous function f is the limit of the sum of the areas of the approximating rectangles

Note: we do NOT specify left/right/midpoint approximation. This is because in the limit all values are equivalent, and equal the area.



Let's look at another function:

divide the interval $[a,b]$ into n equal size intervals

Q: length? A: $\frac{b-a}{n} = \Delta x$

Randomly choose sample pts $f(x_i)$

$$\text{Area} \approx \Delta x \cdot f(x_1) + \Delta x \cdot f(x_2) + \cdots + \Delta x \cdot f(x_3)$$

Summation notation $\sum_{i=1}^{n} f(x_i) \cdot \Delta x = f(x_1)\Delta x + \cdots + f(x_n)\Delta x$

$$\boxed{\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x}$$

Let's Return to the example $f(x) = x^2$ and write the area under the curve from 0 to 1 as a limit.

Q: what is Δx? A: $\frac{1}{n}$

Q: what is $f(x_i)$? A: using right endpts it is $\left(\frac{i}{n}\right)^2$

Thus $\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \cdot \frac{1}{n}$

[similar to 5.1.24] note: Area actually equals this limit.

The Distance Problem: find distance traveled by an object during a given time period if the velocity is known at all times.

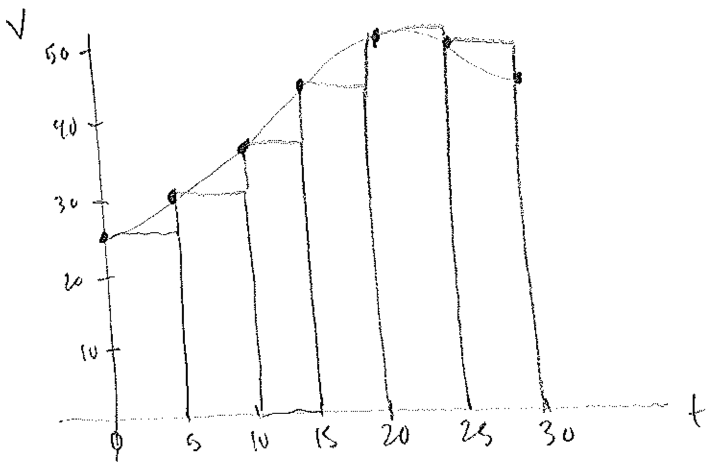
For constant velocity we have distance = velocity x time

Harder if velocity is nonconstant.

ex]

Time	0	5	10	15	20	25	30	seconds
Velocity	25	31	35	43	47	46	41	m/s

Story: odometer is broken and want to know distance traveled. we take speedometer readings every five seconds



Q: how far does the car travel during the time interval 0 to 30

know for constant velocity, $d = v \times t$. So for each 5 second subinterval let's assume constant velocity.

$$D \approx 5s \cdot 25\tfrac{m}{s} + 5s \cdot 31\tfrac{m}{s} + 5s \cdot 35\tfrac{m}{s} + 5s \cdot 43\tfrac{m}{s} + 5s \cdot 47\tfrac{m}{s} + 5s \cdot 46\tfrac{m}{s} = 1135\,m$$

We can do the same with a right endpoint approximation 1215m

Q: How could we produce a more accurate estimate of the distance?

In fact: $D = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{30}{n} \cdot V_i$