Lecture Notes 1/7

Review of Math 1 : Derivatives

- Definition via limits
- How to Calculate
- Applications

Definition via Limits $\qquad f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

How to Calculate

- Product Rule $\quad \frac{d}{dx}\left[f(x) \cdot g(x) \right] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

ex) $\quad \frac{d}{dx}\left[x^2 \cdot \sin x \right] = 2x \cdot \sin x + x^2 \cdot \cos x$

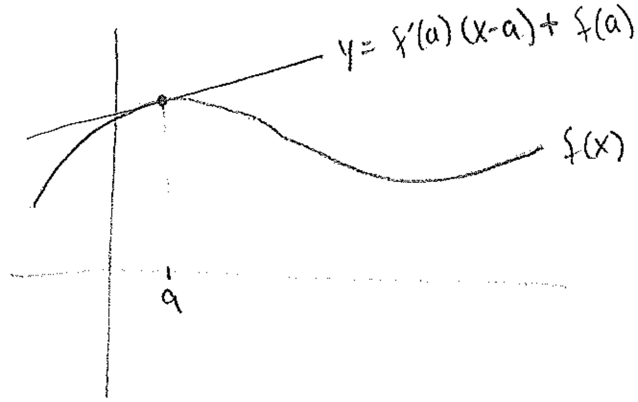
- Chain Rule $\quad \frac{d}{dx}\left[f(g(x)) \right] = f'(g(x)) \cdot g'(x)$

ex) $\quad \frac{d}{dx}\left[\sin(x^2) \right] = \cos(x^2) \cdot 2x$

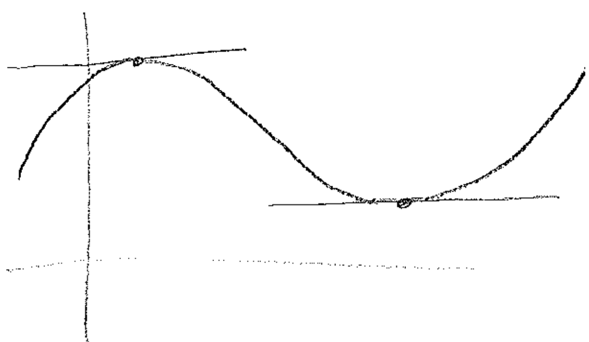
ex) $\quad \frac{d}{dx}\left[e^{\sin x} \right] = e^{\sin x} \cdot \cos x$

Applications

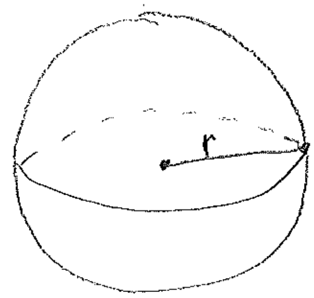
- tangent lines



- maxima and minima via critical points



- Rates of Change : $V = \frac{4}{3}\pi r^3$



If $\frac{dr}{dt} = 2\ m/s$ how fast is the Volume growing when $r = 10\ m$?

$$\frac{dV}{dt} = \frac{4}{3}\pi\, 3r^2 \frac{dr}{dt} = 4\pi\,(10\,m)^2 \cdot 2\ m/s = 800\pi\ \frac{m^3}{s}$$

Math 2: Integrals

Definition via limits $$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

How to Calculate
- substitution
- by parts
- trig methods

Applications: Area, distance, lengths, volume

Antiderivatives: Q: Given $f(x) = x^2 + 5$ does there exist a function $F(x)$ such that $F'(x) = f(x)$?

If so $F(x)$ is called the antiderivative of $f(x)$.

A: In this case, YES, with $F(x) = \frac{x^3}{3} + 5x$

Q: Is this function $F(x)$ unique, i.e. does there exist $G(x) \neq F(x)$ with $G'(x) = f(x)$.

A: YES, let $G(x) = F(x) + C$, where C is any constant.

examples of antiderivatives (see pg 345).

function	antiderivative
x^n	$\frac{x^{n+1}}{n+1}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
e^x	e^x
$\frac{1}{x}$	$\ln \lvert x \rvert$