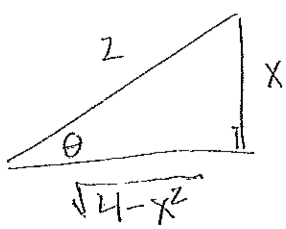
HW6 Solutions

7.3.1 $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}}$ $\qquad x = 2\sin\theta$

$\qquad\qquad dx = 2\cos\theta\, d\theta$

$$= \int \frac{2\cos\theta\, d\theta}{4\sin^2\theta\sqrt{4-4\sin^2\theta}} = \int \frac{2\cos\theta}{4\sin^2\theta \cdot 2\cos\theta}\, d\theta = \frac{1}{4}\int \frac{1}{\sin^2\theta}\, d\theta$$

$= \frac{1}{4}\int \csc^2\theta\, d\theta$ this is a table integral $= \frac{1}{4}(-\cot\theta) + C$



$$= -\frac{1}{4} \cdot \frac{\sqrt{4-x^2}}{x} + C$$

7.3.2 $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\, dx$ $\qquad x = 2\tan\theta$

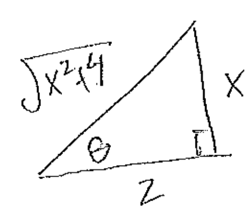
$\qquad\qquad dx = 2\sec^2\theta\, d\theta$

$$= \int \frac{8\tan^3\theta}{\sqrt{4\tan^2\theta+4}} \cdot 2\sec^2\theta\, d\theta = \int \frac{8\tan^3\theta \cdot 2\sec^2\theta}{2\sec\theta}\, d\theta$$

$$= 8\int \tan^3\theta \cdot \sec\theta\, d\theta = 8\int \tan^2\theta \cdot \sec\theta\tan\theta\, d\theta$$

$= 8\int (\sec^2\theta - 1)\cdot \sec\theta\tan\theta\, d\theta \qquad u = \sec\theta \quad du = \sec\theta\tan\theta\, d\theta$

$= 8\int (u^2-1)\, du = 8\left[\frac{u^3}{3} - u\right] + C$

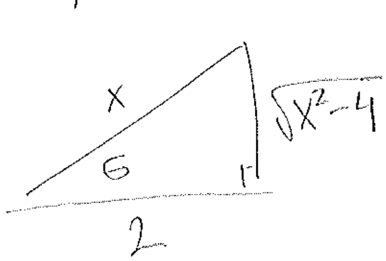


$$= 8\left[\frac{1}{3}\left(\frac{\sqrt{x^2+4}}{2}\right)^3 - \left(\frac{\sqrt{x^2+4}}{2}\right)\right] + C$$

7.3.3 $\displaystyle\int \frac{\sqrt{x^2-4}}{x}\,dx$ $x = 2\sec\theta$ $dx = 2\sec\theta\tan\theta\,d\theta$

$$= \int \frac{\sqrt{4\sec^2\theta - 4}}{2\sec\theta}\, 2\sec\theta\tan\theta\,d\theta = \int \frac{2\tan\theta \cdot 2\sec\theta\tan\theta}{2\sec\theta}\,d\theta$$

$= 2\displaystyle\int \tan^2\theta\,d\theta$ this is a table integral $= 2(\tan\theta - \theta) + C$



$$= 2\left(\frac{\sqrt{x^2-4}}{2} - \operatorname{arcsec}\left(\frac{x}{2}\right)\right) + C$$

$x = \sin\theta$ $dx = \cos\theta\,d\theta$

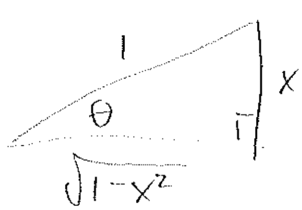
7.3.4 $\displaystyle\int_0^1 x^3\sqrt{1-x^2}\,dx$

$$= \int_0^1 \sin^3\theta\sqrt{1-\sin^2\theta}\cdot\cos\theta\,d\theta = \int_0^1 \sin^3\theta\cdot\cos^2\theta\,d\theta$$

$$= \int_0^1 \sin^2\theta\cos^2\theta\sin\theta\,d\theta = \int_0^1 (1-\cos^2\theta)\cos^2\theta\sin\theta\,d\theta$$

$u = \cos\theta$
$du = -\sin\theta\,d\theta$
$$= -\int_0^1 (1-u^2)u^2\,du = -\int_0^1 u^2 - u^4\,du$$

$$= \frac{u^5}{5} - \frac{u^3}{3}\Bigg|_{x=0}^{x=1}$$



$$= \frac{1}{5}\left(\sqrt{1-x^2}\right)^5 - \frac{1}{3}\left(\sqrt{1-x^2}\right)^3\Bigg|_0^1$$

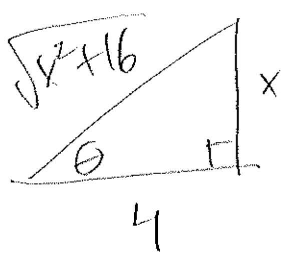
$$= \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

7.3.9 $\displaystyle\int \frac{dx}{\sqrt{x^2+16}}$ $\qquad x = 4\tan\theta$

$\qquad dx = 4\sec^2\theta \, d\theta$

$$= \int \frac{4\sec^2\theta}{\sqrt{16\tan^2\theta+16}}\,d\theta = \int \frac{4\sec^2\theta}{4\sec\theta}\,d\theta = \int \sec\theta \, d\theta$$

this is a table integral $= \ln\left|\sec\theta + \tan\theta\right| + C$



$$= \ln\left|\frac{\sqrt{x^2+16}}{4} + \frac{x}{4}\right| + C$$

$$= \ln\left|\frac{\sqrt{x^2+16} + x}{4}\right| + C$$

$$= \ln\left|\sqrt{x^2+16} + x\right| - \ln(4) + C$$

$$= \ln\left|\sqrt{x^2+16} + x\right| + K$$

where K is a new constant, $K = C - \ln(4)$

note: the solution $\ln\left|\frac{\sqrt{x^2+16}}{4} + \frac{x}{4}\right| + C$ is correct.

7.4.10 $\displaystyle\int \frac{y}{(y+4)(2y-1)}\,dy$

$$\frac{y}{(y+4)(2y-1)} = \frac{A}{y+4} + \frac{B}{2y-1}$$

$$y = A(2y-1) + B(y+4)$$

$$y = (2A+B)y + (-A+4B)1$$

$2A+B=1$ $\qquad$ $-A+4B=0$

$8B+B=1$ $\qquad$ $A=4B$

$B=1/9$ $\qquad$ $A=4/9$

$$= \int \frac{4/9}{y+4} + \frac{1/9}{2y-1}\,dy = \frac{4}{9}\ln(y+4) + \frac{1}{9}\cdot\frac{1}{2}\ln(2y-1) + C$$

7.4.12 $\displaystyle\int_0^1 \frac{x-4}{x^2-5x+6}\,dx = \int_0^1 \frac{x-4}{(x-3)(x-2)}\,dx$

$$\frac{x-4}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$$

$$x-4 = A(x-2) + B(x-3)$$

$$x-4 = (A+B)\,x + (-2A-3B)\cdot 1$$

$A+B=1$ $\qquad -2A-3B=-4$

$A=1-B$ $\qquad 2A+3B=4$

$\qquad\qquad 2(1-B)+3B=4$

$A=-1$ $\qquad 2-2B+3B=4$

$\qquad\qquad B=2$

$$\int_0^1 \frac{-1}{x-3}+\frac{2}{x-2}\,dx = -\ln|x-3| + 2\ln|x-2| \Big|_0^1$$

$$= -\ln|-2| + 2\ln|-1| - \left(-\ln|-3| + 2\ln|-2|\right)$$

$$= -3\ln(2) + \ln(3) = \ln(3/8)$$

7.4.22 $\displaystyle\int \frac{ds}{s^2(s-1)^2}$

$$\frac{1}{s^2(s-1)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{D}{(s-1)^2}$$

$$1 = A(s\cdot(s-1)^2) + B(s-1)^2 + C(s^2\cdot(s-1)) + D\cdot s^2$$

$$1 = A(s(s^2-2s+1)) + B(s^2-2s+1) + C(s^3-s^2) + Ds^2$$

$$1 = A(s^3-2s^2+s) + B(s^2-2s+1) + C(s^3-s^2) + Ds^2$$

$$1 = (A+C)s^3 + (-2A+B-C+D)s^2 + (A-2B)s + B\cdot 1$$

$A+C=0$ $\quad -2A+B-C+D=0$ $\quad A-2B=0$ $\quad B=1$

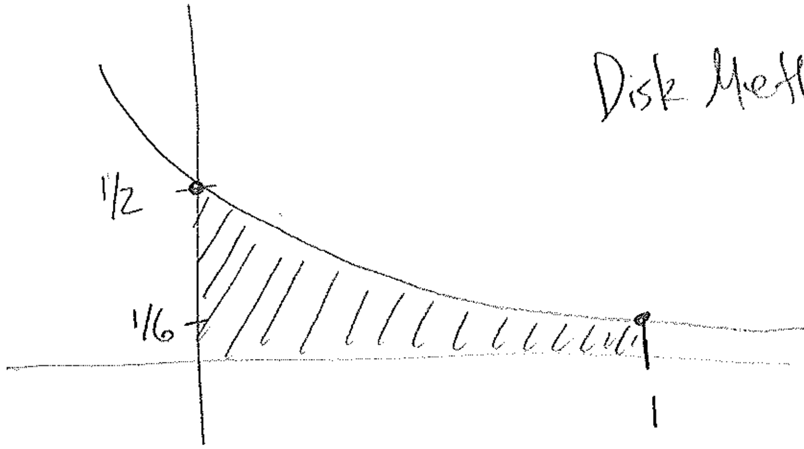
$C=-2$ $\quad D=C-B+2A$ $\quad A=2$

$D=1$

$$\int \frac{2}{s} + \frac{1}{s^2} - \frac{2}{s-1} + \frac{1}{(s-1)^2}\, ds$$

$$= 2\ln(s) - \frac{1}{s} - 2\ln(s-1) - \frac{1}{s-1} + C$$

7.4.66



Disk Method for (a)

a) find volume by rotating region about x-axis

$$Vol = \int_0^1 \pi \left(\frac{1}{x^2+3x+2}\right)^2 dx = \pi \int_0^1 \frac{1}{(x+1)^2 (x+2)^2} dx$$

$$\frac{1}{(x+1)^2 (x+2)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2} + \frac{D}{(x+2)^2}$$

$$1 = A(x+1)(x+2)^2 + B(x+2)^2 + C(x+1)^2(x+2) + D(x+1)^2$$

$$1 = A(x+1)(x^2+4x+4) + B(x^2+4x+4) + C(x^2+2x+1)(x+2) + D(x^2+2x+1)$$

$$1 = A(x^3+5x^2+8x+4) + B(x^2+4x+4) + C(x^3+4x^2+5x+2) + D(x^2+2x+1)$$

$$1 = (A+C)x^3 + (5A+B+4C+D)x^2 + (8A+4B+5C+2D)x$$
$$+ (4A+4B+2C+D)\cdot 1$$

$A+C=0 \quad 5A+B+4C+D=0 \quad 8A+4B+5C+2D=0$

$4A+4B+2C+D=1$

$-A = C \qquad A+B+D=0 \qquad 3A+4B+2D=0 \qquad 2A+4B+D=1$

$A+B=-D \qquad 3A+4B-2A-2B=0 \qquad 2A+4B-A-B=1$

$A+2B=0 \qquad A+3B=1$

$A=-2B \qquad B=1$

$C=2 \qquad D=1 \qquad A=-2$

$$\pi \int_0^1 \frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{2}{(x+2)} + \frac{1}{(x+2)^2}\, dx$$

$$= \pi \left[-2\ln(x+1) - \frac{1}{x+1} + 2\ln(x+2) - \frac{1}{x+2} \right]_0^1$$

$$= \pi \left[-2\ln(2) - \frac{1}{2} + 2\ln(3) - \frac{1}{3} \right] - \pi \left[0 - 1 + 2\ln(2) - \frac{1}{2} \right]$$

$$= \pi \left[-4\ln(2) + 2\ln(3) + \frac{2}{3} \right]$$

7.4.66 Cylindrical Shells for (b)

b) find volume by rotating about y-axis

$$\text{Vol} = \int_0^1 2\pi x \cdot \frac{1}{x^2+3x+2}\,dx = 2\pi \int_0^1 \frac{x}{(x+2)(x+1)}\,dx$$

$$\frac{x}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$$

$$x = A(x+1) + B(x+2)$$

$$x = (A+B)x + (A+2B)\cdot 1$$

$A+B=1$ $\quad$ $A+2B=0$

$A=1-B$ $\quad$ $1-B+2B=0$

$B=-1$

$A=2$

$$\text{Vol} = 2\pi \int_0^1 \frac{2}{x+2} - \frac{1}{x+1}\,dx = 2\pi\left(2\ln(x+2) - \ln(x+1)\right)\Big|_0^1$$

$$= 2\pi\left(2\ln(3) - \ln(2)\right) - 2\pi\left(2\ln(2) - 0\right)$$

$$= 2\pi\left(2\ln(3) - 3\ln(2)\right)$$