HW5 Solutions,

6.5.4 $\text{Avg} = \frac{1}{3-1}\int_1^3 \frac{x}{\sqrt{3+x^2}}\,dx$ $\qquad u = 3+x^2$, $du = 2x\,dx$

$$= \frac{1}{3-1}\cdot\frac{1}{2}\int_{x=1}^{x=3}\frac{1}{\sqrt{u}}\,du = \frac{1}{3-1}\cdot\frac{1}{2}\cdot 2\sqrt{u}\,\Big|_{x=1}^{x=3}$$

$$= \frac{1}{2}\sqrt{3+x^2}\,\Big|_1^3 = \frac{\sqrt{12}}{2} - 1$$

6.5.14 We need to solve the below equation for b:

$$3 = \frac{1}{b}\int_0^b 2+6x-3x^2\,dx$$

$$= \frac{1}{b}\left(2x+3x^2-x^3\right)\Big|_0^b = \frac{1}{b}\left(2b+3b^2-b^3\right) = 2+3b-b^2$$

we have $3 = 2+3b-b^2$ so that $0 = -1+3b-b^2$

Now use quadratic formula

$$b = \frac{-3\pm\sqrt{9-4}}{-2} \qquad b \approx \{2.6 \text{ and } 0.4\}$$

6.5.17 $T(t) = 50 + 14\sin\left(\frac{\pi t}{12}\right)$

where $T(0) =$ temp at 9am and

$T(12) =$ temp at 9pm

Thus Avg temperature between 9am and 9pm is given by:

$$\text{Avg} = \frac{1}{12-0}\int_0^{12} 50 + 14\sin\left(\frac{\pi t}{12}\right)\,dt = \frac{1}{12}\left[50t - 14\cdot\frac{12}{\pi}\cos\left(\frac{\pi t}{12}\right)\right]\Bigg|_0^{12}$$

$$= \frac{1}{12}\left[50\cdot 12 - 14\cdot\frac{12}{\pi}\cos(\pi)\right] - \frac{1}{12}\left[0 - 14\cdot\frac{12}{\pi}\cos(0)\right]$$

$$= \frac{1}{12}\left[50\cdot 12 + 14\cdot\frac{12}{\pi}\right] + \frac{1}{12}\left[14\cdot\frac{12}{\pi}\right] \approx 59^\circ$$

7.1.12 $\int p^5 \cdot \ln(p)\,dp$

$u = \ln p \qquad dv = p^5\,dp$

$du = 1/p\,dp \qquad v = \frac{1}{6}p^6$

$$= \frac{1}{6}p^6\cdot\ln p - \int \frac{1}{6}\cdot\frac{1}{p}\cdot p^6\,dp = \frac{1}{6}p^6\ln p - \frac{1}{6}\int p^5\,dp$$

$$= \frac{1}{6}p^6\ln p - \frac{1}{36}p^6 + C$$

7.1.18 $\int e^{-\theta} \cos 2\theta \, d\theta$

note: this problem is similar to $\int e^{x} \cdot \sin x \, dx$

$u = e^{-\theta}$ $\quad dv = \cos 2\theta \, d\theta$

$du = -e^{-\theta} \, d\theta$ $\quad v = \frac{1}{2} \sin 2\theta$

$$= e^{-\theta} \cdot \frac{1}{2} \sin 2\theta - \int \frac{1}{2} \sin 2\theta \cdot (-e^{-\theta}) \, d\theta$$

$$= e^{-\theta} \cdot \frac{1}{2} \sin 2\theta + \frac{1}{2} \int e^{-\theta} \sin 2\theta \, d\theta$$

$u = e^{-\theta}$ $\quad dv = \sin 2\theta \, d\theta$

$du = -e^{-\theta} \, d\theta$ $\quad v = -\frac{1}{2} \cos 2\theta$

$$= e^{-\theta} \cdot \frac{1}{2} \sin 2\theta + \frac{1}{2} \left[e^{-\theta} \cdot \frac{-1}{2} \cos 2\theta - \int \frac{1}{2} \cos 2\theta \cdot e^{-\theta} \, d\theta \right]$$

therefore we have

$$\int e^{-\theta} \cos 2\theta \, d\theta = e^{-\theta} \cdot \frac{1}{2} \sin 2\theta - \frac{1}{4} e^{-\theta} \cos 2\theta - \frac{1}{4} \int e^{-\theta} \cos 2\theta \, d\theta$$

$$\frac{5}{4} \int e^{-\theta} \cos 2\theta \, d\theta = e^{-\theta} \cdot \frac{1}{2} \sin 2\theta - \frac{1}{4} e^{-\theta} \cos 2\theta$$

$$\int e^{-\theta} \cos 2\theta \, d\theta = \frac{4}{5} \left[e^{-\theta} \cdot \frac{1}{2} \sin 2\theta - \frac{1}{4} e^{-\theta} \cos 2\theta \right] + C$$

7.1.38 $\int t^3 e^{-t^2}\,dt$ $\qquad u=-t^2 \quad du=-2t\,dt$

$-\frac{1}{2}du = t\,dt$

$= \int t^2 \cdot e^{-t^2} \cdot t\,dt = \frac{-1}{2}\int (-u)e^u\,du = \frac{1}{2}\int ue^u\,du$

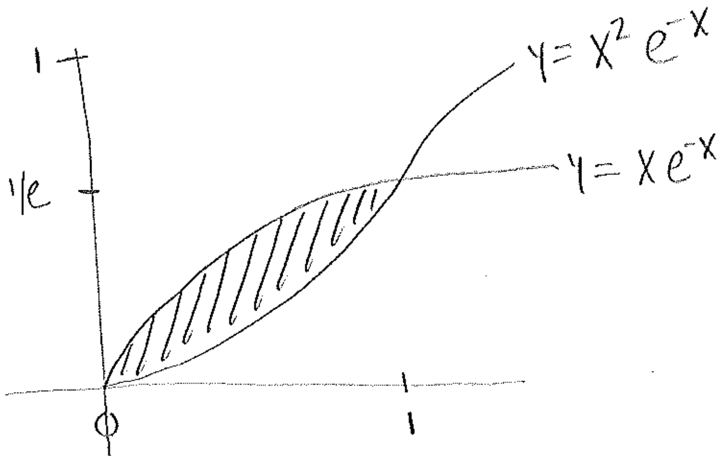
Now let $\quad u=u \quad dv = e^u\,du$

$du = du \quad v = e^u$

$= \frac{1}{2}\left[ue^u - \int e^u\,du\right] = \frac{1}{2}\left[ue^u - e^u\right]$

$= \frac{1}{2}\left[-t^2 e^{-t^2} - e^{-t^2}\right] + C$

7.1.58 Area between curves: start by drawing picture



the curves cross at $(0,0)$ and $(1, 1/e)$

on the interval $[0,1]$ the curve $y = xe^{-x}$ is on top.

7.1.58 continued

$$\text{Area} = \int_0^1 xe^{-x} - x^2e^{-x}\,dx = \int_0^1 xe^{-x}\,dx - \int_0^1 x^2e^{-x}\,dx$$

use integration by parts on the second integral:

$u = x^2 \qquad dv = e^{-x}\,dx$

$du = 2x\,dx \qquad v = -e^{-x}$

$$= \int_0^1 xe^{-x}\,dx - \left[-x^2e^{-x} + \int_0^1 e^{-x}\,2x\,dx\right]$$

$$= x^2e^{-x} - \int_0^1 xe^{-x}\,dx \qquad \begin{matrix} u = x & dv = e^{-x}\,dx \\ du = dx & v = -e^{-x} \end{matrix}$$

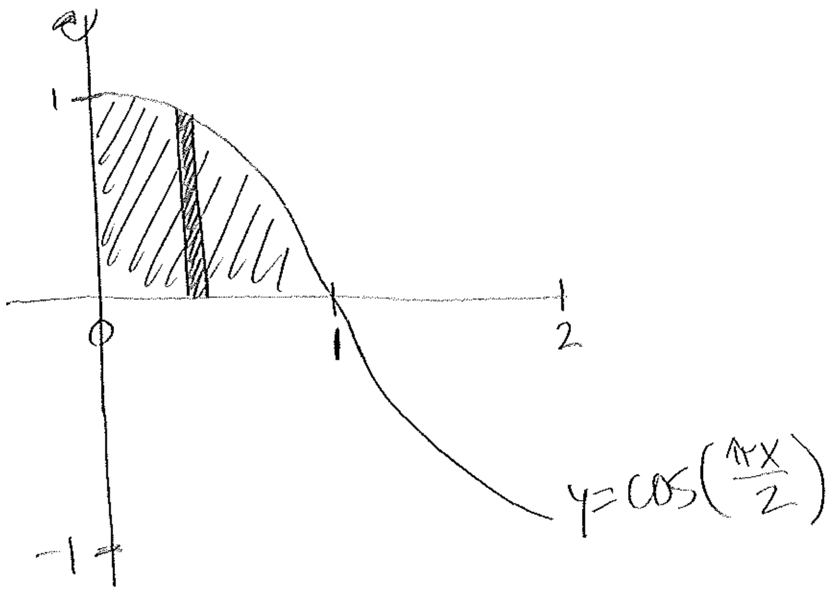
$$= x^2e^{-x} - \left[-xe^{-x} + \int_0^1 e^{-x}\,dx\right]$$

$$= x^2e^{-x} + xe^{-x} + e^{-x}\Big|_0^1$$

$$= e^{-1} + e^{-1} + e^{-1} - 1 = \frac{1}{3e} - 1 \approx .10$$

this answer makes sense because it is positive and the shaded region is about 1/10 of the unit square

7.1.61



by parts

$u = x \qquad dv = \cos\left(\frac{\pi x}{2}\right)dx$

$du = dx \qquad v = \frac{2}{\pi}\sin\left(\frac{\pi x}{2}\right)$

$$\text{Vol} = \int_0^1 2\pi x \cdot \cos\left(\frac{\pi x}{2}\right) dx$$

$$= 2\pi\left[x \cdot \frac{2}{\pi}\sin\left(\frac{\pi x}{2}\right) - \int_0^1 \frac{2}{\pi}\sin\left(\frac{\pi x}{2}\right)dx\right]$$

$$= 4x \cdot \sin\left(\frac{\pi x}{2}\right) + 4 \cdot \cos\left(\frac{\pi x}{2}\right) \cdot \frac{2}{\pi} \Bigg|_0^1$$

$$= 4 + 0 - \left(0 + 4 \cdot \frac{2}{\pi}\right) = 4 - 8/\pi$$

7.1.67 note: $v(t) = t^2 e^{-t} \geq 0$ for $t \geq 0$

$$\text{distance} = \int_0^t |v(t)|\,dt = \int_0^t t^2 e^{-t}\,dt$$

$u = t^2 \quad dv = e^{-t}\,dt$

$du = 2t\,dt \quad v = -e^{-t}$

$$= -t^2 e^{-t} + \int e^{-t} \cdot 2t\,dt$$

$u = 2t \quad dv = e^{-t}\,dt$

$du = 2\,dt \quad v = -e^{-t}.$

$$= -t^2 e^{-t} + 2t(-e^{-t}) + \int 2e^{-t}\,dt$$

$$= -t^2 e^{-t} - 2te^{-t} - 2e^{-t} \Big|_0^t$$

$$= -t^2 e^{-t} - 2te^{-t} - 2e^{-t} - (-2)$$

$$= 2 - e^{-t}(t^2 + 2t + 2)$$

7.2.2 $\int \sin^3\theta \cos^4\theta \, d\theta = \int \sin^2\theta \cos^4\theta \sin\theta \, d\theta$

$u = \cos\theta$
$du = -\sin\theta \, d\theta$

$= \int (1-\cos^2\theta)\cos^4\theta \sin\theta \, d\theta$

$= -\int (1-u^2)\cdot u^4 \, du = -\int u^4 - u^6 \, du = -\frac{1}{5}u^5 + \frac{1}{7}u^7$

$= -\frac{1}{5}\cos^5\theta + \frac{1}{7}\cos^7\theta + C$

7.2.17 $\int \cos^2 x \cdot \tan^3 x \, dx = \int \cos^2 x \cdot \frac{\sin^3 x}{\cos^3 x} \, dx$

$= \int \frac{\sin^3 x}{\cos x} \, dx = \int \frac{\sin^2 x}{\cos x} \cdot \sin x \, dx = \int \frac{1-\cos^2 x}{\cos x} \cdot \sin x \, dx$

$u = \cos x$
$du = -\sin x \, dx$

$= -\int \frac{1-u^2}{u} \, du = -\int \frac{1}{u} - u \, du$

$= -\left(\ln u - \frac{u^2}{2}\right) = \frac{\cos^2 x}{2} - \ln(\cos x) + C$

7.2.24

$$\int \tan^2 x + \tan^4 x \, dx = \int \tan^2 x (1 + \tan^2 x) \, dx$$

$$= \int \tan^2 x \cdot \sec^2 x \, dx \qquad \begin{aligned} u &= \tan x \\ du &= \sec^2 x \, dx \end{aligned}$$

$$= \int u^2 \, du = \frac{u^3}{3} + C = \frac{1}{3} \tan^3 x + C$$