5.3.4

a) $g(0) = \int_0^0 f(t)\,dt = 0$ $\qquad g(6) = \int_0^6 f(t)\,dt = 0$ b/c positive and negative areas cancel.

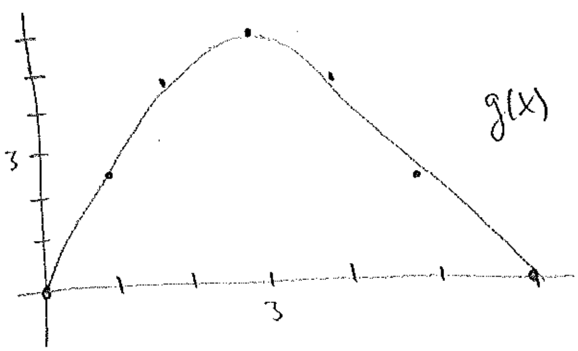
b) $g(1) = 2.7 \qquad g(2) = 4.7 \qquad g(3) = 5.5$

$g(4) = 4.7 \qquad g(5) = 2.7$

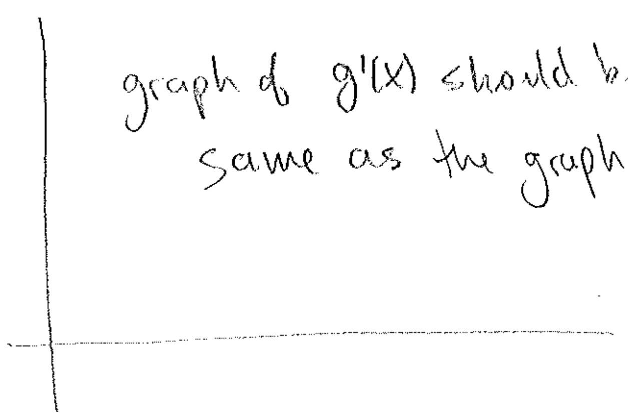
c) g increasing on $[0, 3]$

d) g has max at $x = 3$

e)



f)



graph of $g'(x)$ should be the same as the graph of f.

5.3.8

$$\frac{d}{dx} g(x) = \frac{d}{dx}\int_3^x e^{t^2-t} = \boxed{e^{x^2-x}}$$ by FTC, Part 1.

5.3.16 let $u = x^4$

$$\frac{d}{dx}\int_0^{x^4} \cos^2\theta \, d\theta = \frac{d}{du}\left[\int_0^u \cos^2\theta \, d\theta\right]\frac{du}{dx} = \cos^2 u \cdot \frac{du}{dx} = \cos^2(x^4)\cdot 4x^3$$

Alternately let $g(x) = \int_0^x \cos^2\theta \, d\theta$ and $u(x) = x^4$.

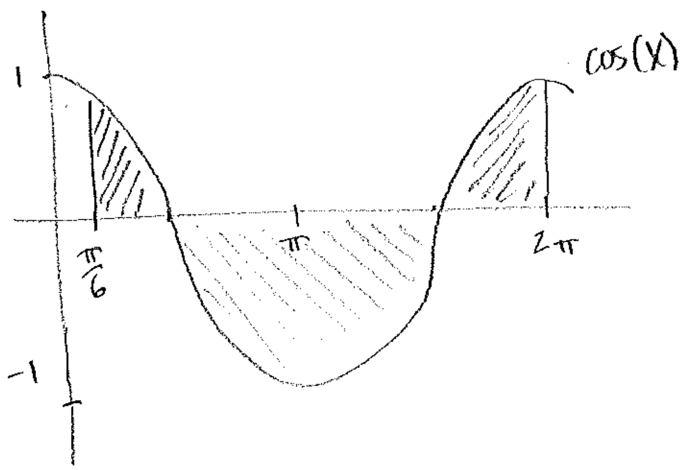
Then $\frac{d}{dx}\int_0^{x^4} \cos^2\theta \, d\theta = \frac{d}{dx} g(u(x)) = g'(u(x))\cdot u'(x) = \boxed{\cos^2(x^4)\cdot 4x^3}$

5.3.24

$$\int_1^8 x^{-2/3}\, dx = 3x^{1/3}\Big|_1^8 = 3\cdot 2 - 3\cdot 1 = \boxed{3}$$

5.3.54

$$\int_{\pi/6}^{2\pi} \cos x \, dx = \sin x \Big|_{\pi/6}^{2\pi} = \sin 2\pi - \sin \pi/6 = 0 - \frac{1}{2} = \boxed{-\frac{1}{2}}$$



$\int_{\pi/6}^{2\pi} \cos x \, dx$ is the area above the x-axis shaded in the picture minus the area below the x-axis shaded in the picture.

5.3.69

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \cdot \frac{1}{n} = \int_0^1 x^3 \, dx = \frac{x^4}{4}\Big|_0^1 = \boxed{\frac{1}{4}}$$

5.4.10

$$\int v(v^2+2)^2 \, dv = \int v(v^4+4v^2+4)\, dv = \int v^5 + 4v^3 + 4v \, dv$$

$$= \boxed{\frac{v^6}{6} + v^4 + 2v^2 + C}$$

AWZ #4

5.4.12

$$\int x^2 + 1 + \frac{1}{x^2+1}\,dx = \int x^2\,dx + \int 1\,dx + \int \frac{1}{x^2+1}\,dx$$

$$= \boxed{\frac{x^3}{3} + x + \arctan x + C}$$

5.4.32

$$\int_1^4 \frac{\sqrt{y} - y}{y^2}\,dy = \int_1^4 y^{-3/2} - y^{-1}\,dy = -2y^{-1/2} - \ln y \Big|_1^4$$

$$= \frac{-2}{\sqrt{4}} - \ln 4 - \left(\frac{-2}{1} - \ln 1\right) = -1 - \ln 4 + 2 = \boxed{1 - \ln 4}$$

HW 2 #5

5.4.60

$v(t) = t^2 - 2t - 8 \qquad 1 \le t \le 6$. Let $s(t)$ the position function.

a) displacement is given by $s(6) - s(1) = \int_1^6 v(t)\,dt = \int_1^6 t^2 - 2t - 8\,dt$

$$= \frac{t^3}{3} - t^2 - 8t \Big|_1^6 = \frac{6^3}{3} - 6^2 - 8\cdot 6 - \left(\frac{1}{3} - 1 - 8\right) = \boxed{\frac{-10}{3}}$$

b) distance traveled: we need to know when the velocity is positive vs. negative. We have $v(1) = -9 \qquad v(6) = 16$

Velocity is zero: $0 = t^2 - 2t - 8 = (t-4)(t+2) \Rightarrow t = \{-2, 4\}$

thus velocity is positive on interval $(4, 6]$ and negative on the interval $[1, 4)$.

Total distance $= \int_1^6 |v(t)|\,dt = \int_1^4 -v(t)\,dt + \int_4^6 v(t)\,dt$

$$= -\frac{t^3}{3} + t^2 + 8t \Big|_1^4 + \frac{t^3}{3} - t^2 - 8t \Big|_4^6$$

$$= \frac{-4^3}{3} + 4^2 + 8\cdot 4 - \left(\frac{-1}{3} + 1 + 8\right) + \frac{6^3}{3} - 6^2 - 8\cdot 6 - \left(\frac{4^3}{3} - 4^2 - 4\cdot 8\right) = \boxed{\frac{98}{3}}$$

HW2
#6

5.4.62

$a(t) = 2t+3$, $v(0) = -4$ $0 \le t \le 3$

a) In general, $v(t) = \int a(t)\,dt = \int 2t+3\,dt = t^2+3t+C$

Now $v(0) = 0^2+3(0)+C = C$ hence $C=-4$ and

$\boxed{v(t) = t^2+3t-4.}$ This is velocity at time t.

b) distance traveled: we need to know the intervals on which $v(t)$ is positive vs. negative. $v(0) = -4$ and $v(3) = 14$

$0 = t^2+3t-4 = (t+4)(t-1)$ hence v is zero at $t = \{1, -4\}$

Thus, $v(t)$ positive on interval $(1,3]$ and negative on $[0,1)$.

Total distance $= \int_0^3 |v(t)|\,dt = \int_0^1 -v(t)\,dt + \int_1^3 v(t)\,dt$

$= -\frac{t^3}{3} - \frac{3}{2}t^2 + 4t\Big|_0^1 + \frac{t^3}{3} + \frac{3}{2}t^2 - 4t\Big|_1^3$

$= -\frac{1}{3} - \frac{3}{2} + 4 - (0) + \frac{27}{3} + \frac{3}{2}\cdot 9 - 4\cdot 3 - \left(\frac{1}{3} + \frac{3}{2} - 4\right) = \boxed{\frac{89}{6}}$