Last time: Improper Integrals $\int_a^{\infty} f(x)\,dx \quad \int_{-\infty}^{a} f(x)\,dx \quad \int_{-\infty}^{\infty} f(x)\,dx$

ex) $\int_1^{\infty} \frac{1}{x^2}\,dx = \lim_{t\to\infty} \int_1^{t} \frac{1}{x^2}\,dx = \lim_{t\to\infty} \frac{-1}{x}\Big|_1^t = \lim_{t\to\infty} \frac{-1}{t} + 1 = 1$

This area is actually finite.

ex) $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = \int_{-\infty}^{0} \frac{1}{1+x^2}\,dx + \int_0^{\infty} \frac{1}{1+x^2}\,dx$

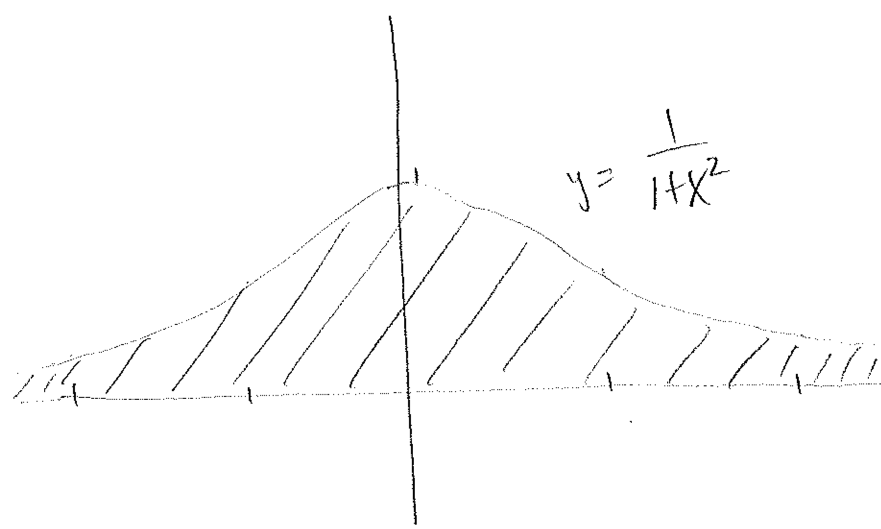
$\int_0^{\infty} \frac{1}{1+x^2}\,dx = \lim_{t\to\infty} \int_0^{t} \frac{1}{1+x^2}\,dx = \lim_{t\to\infty} \arctan x \Big|_0^t = \lim_{t\to\infty} [\arctan t - \arctan 0]$

$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$\int_{-\infty}^{0} \frac{1}{1+x^2}\,dx = \lim_{t\to-\infty} \int_t^{0} \frac{1}{1+x^2}\,dx = \lim_{t\to-\infty} \arctan x \Big|_t^0 = \lim_{t\to-\infty} [\arctan 0 - \arctan t]$

$= 0 - \left(-\frac{\pi}{2}\right) = \frac{\pi}{2}$

Thus $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = \frac{\pi}{2} + \frac{\pi}{2} = \pi$



ex1 (see 7.8.20) L'hopital's Rule

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$$\int_{-\infty}^{0} xe^x \, dx = \lim_{t \to -\infty} \int_t^0 xe^x \, dx$$

Integral: $\int_t^0 xe^x \, dx = xe^x - \int e^x \, dx = xe^x - e^x \Big|_t^0 = -1 - (te^t - e^t)$

$$\int_{-\infty}^{0} xe^x \, dx = \lim_{t \to -\infty} -1 - te^t + e^t = -1 - \frac{-\infty}{\infty} + \frac{1}{\infty}$$ Indeterminent form

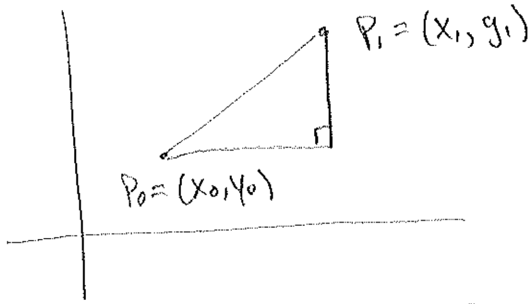
Need to apply L'hopital's Rule

$$\lim_{t \to -\infty} te^t = \lim_{t \to -\infty} \frac{t}{e^{-t}} \stackrel{\downarrow}{=} \lim_{t \to -\infty} \frac{1}{-e^{-t}} = \frac{1}{-\infty} = 0$$

Thus $\int_{-\infty}^{0} xe^x \, dx = \lim_{t \to -\infty} -1 - te^t + e^t = -1 + 0 + 0 = -1$

§8.1 Arclength

Recall: distance formula

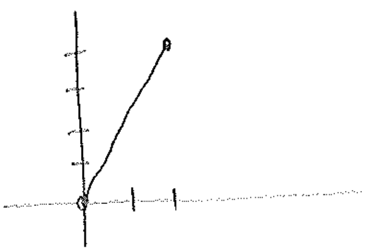


Q: What is distance between P_0 and P_1? $\quad d(P_0, P_1)$

$$d(P_0,P_1) = \sqrt{(x_1-x_0)^2 + (y_1-y_0)^2} \quad \text{via Pythagoras}$$

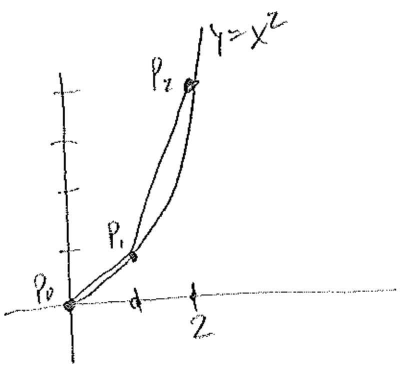
ex) distance between $(0,0)$ and $(2,4)$?

$$d = \sqrt{(2-0)^2 + (4-0)^2} = \sqrt{4+16} = \sqrt{20}$$



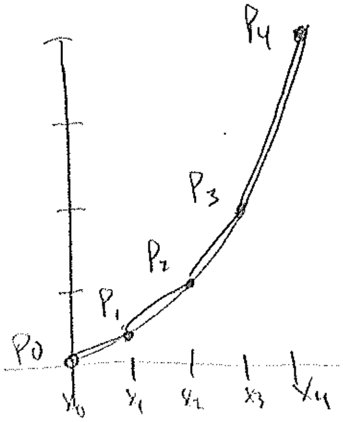
Q: What happens if we have a curvy path

Q: greater OR less than $\sqrt{20}$



length $\approx d(P_0,P_1) + d(P_1,P_2)$

note: $P_0 = (x_0, f(x_0)) \quad P_1 = (x_1, f(x_1))$ etc...



length $\approx d(P_0,P_1) + d(P_1,P_2) + d(P_2,P_3) + d(P_3,P_4)$

$$\text{length} \approx \sum_{i=1}^{n} d(P_i, P_{i-1})$$

$$\text{Arclength} = \lim_{n\to\infty} \sum_{i=1}^{n} d(P_i, P_{i-1})$$