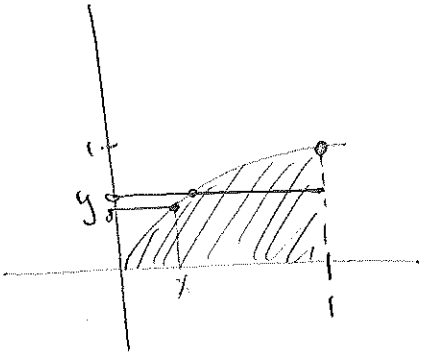
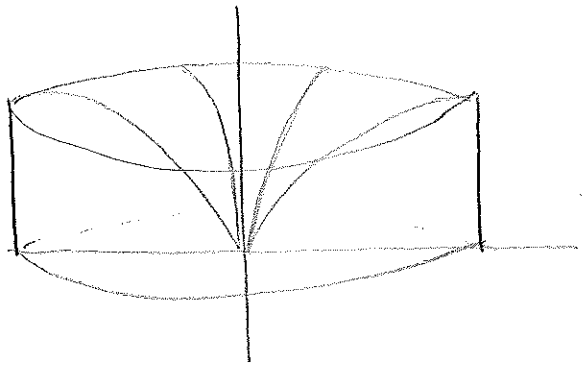
Midterm Review

Volume:

ex) find volume of solid obtained by rotating around the y-axis the region below the curve $y = \sqrt{x}$ between $x=0$ and $x=1$





Washer method:

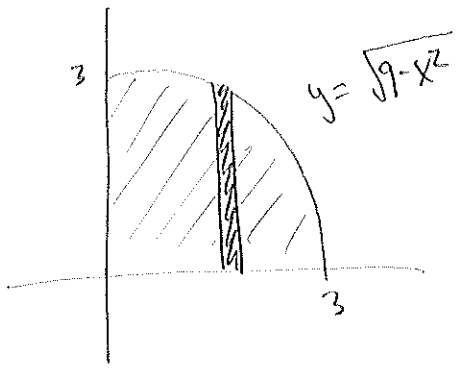
inner radius: $X = y^2$

outer radius: 1

$$A(y) = \pi(1^2) - \pi(y^2)^2$$
$$= \pi(1 - y^4)$$

$$Vol = \int_0^1 \pi(1-y^4)\,dy = \pi\left(y - \frac{1}{5}y^5\right)\Big|_0^1 = \pi\left(1 - \frac{1}{5}\right)$$

ex) Volume of Sphere via Cylindrical Shells



$$Vol = 2\int_0^3 2\pi x \cdot \sqrt{9-x^2}\, dx$$

then u-sub.

Integration

- sit and think : $\int x^2+3x+7\ dx$ $\int \frac{1}{x^2}\ dx$ $\int x^{-1/2}\ dx$

- u-sub : $\int x\sqrt{9-x^2}\, dx$ $\int x \cdot \sin(x^2+1)\ dx$

$\int \frac{x}{\sqrt{9-x^2}}\ dx$ but NOT $\int \frac{\sqrt{9-x^2}}{x}\ dx$

- by parts : $\int u\,dv = uv - \int v\,du$

$\int x^2 \ln x\ dx$ $\int x \cdot \cos x\ dx \neq \frac{x^2}{2} \cdot \sin x$

weird by parts: $\int e^x \cdot \sin x\,dx$ $\int \arctan x\,dx$ $\int \arcsin x\,dx$

$\int \ln x\,dx$

- trig integrals: $\int \sin^m x \cdot \cos^n x\,dx$ and $\int \tan^m x \cdot \sec^n x\,dx$

- trig sub: $\int \frac{\sqrt{9-x^2}}{x}\,dx$ $\int \sqrt{9-x^2}\,dx$

Common mistakes: $+C$

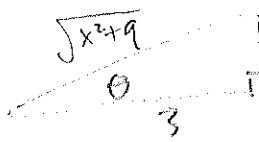
- definite integral by parts. $\int_1^2 x^2 \cdot \ln x\,dx$

  need to evaluate the entire antiderivative

- if $x = 3\tan\theta$

  and need $\sin\theta$ in terms of x at the end,

  NOT okay to write $\sin(\arctan(\frac{x}{3}))$ domain issues

need 

$\sin\theta = \frac{x}{\sqrt{9+x^2}}$