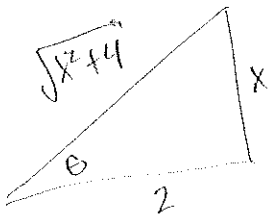
ex) $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$ $\quad x = 2\tan\theta \quad dx = 2\sec^2\theta\,d\theta$

$\sqrt{x^2+4} = \sqrt{4\tan^2\theta+4} = \sqrt{4(\tan^2\theta+1)} = \sqrt{4\sec^2\theta} = 2\sec\theta$

$\int \frac{1}{x^2\sqrt{x^2+4}}\,dx = \int \frac{1}{4\tan^2\theta\cdot 2\sec\theta}\cdot 2\sec^2\theta\,d\theta = \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$

$= \frac{1}{4}\int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\,d\theta = \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$ $\quad u = \sin\theta$, $du = \cos\theta\,d\theta$

$= \frac{1}{4}\int \frac{1}{u^2}\,du = \frac{1}{4}\left(\frac{-1}{u}\right) = \frac{-1}{4}\cdot\frac{1}{\sin\theta} = \frac{-1}{4}\cdot\frac{\sqrt{x^2+4}}{x} + C$

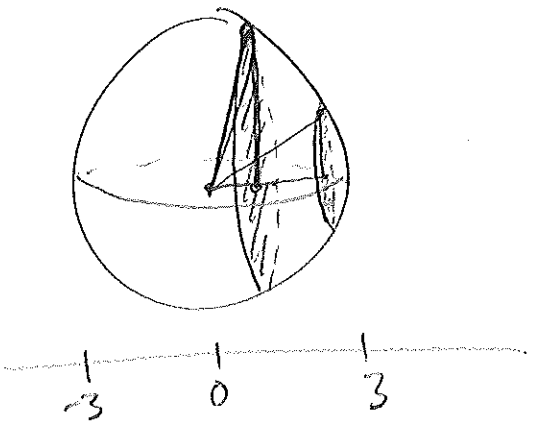


$\sin\theta = \frac{x}{\sqrt{x^2+4}}$

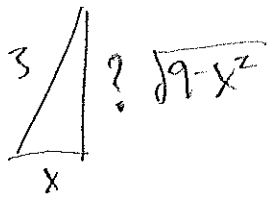
Table of Trig Sub

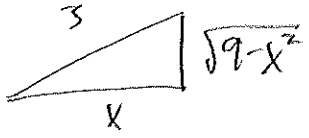
expression	substitution	Pythagorus
$\sqrt{a^2-x^2}$	$x = a\sin\theta$	$1-\sin^2\theta = \cos^2\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$	$1+\tan^2\theta = \sec^2\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

Vol (Sphere)



$$Vol = \int_{-3}^{3} A(x)\,dx$$



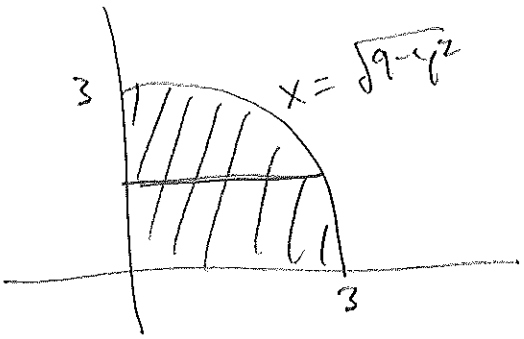


$$A(x) = \pi(9-x^2)$$

$$A(1.5) = \pi(9-1.5^2) > \pi(9-2.9^2) = A(2.9)$$

$$Vol = \int_{-3}^{3} \pi(9-x^2)\,dx$$

Disks: radius $= \sqrt{9-y^2}$



$$A(y) = \pi(9-y^2)$$

$$Vol(Sphere) = 2\int_{0}^{3} \pi(9-y^2)\,dy$$