## §7.2 Trig Integration

| | odd sin | odd cos | even sec | odd tan |
|---|---|---|---|---|
| save | $\sin x$ | $\cos x$ | $\sec^2 x$ | $\sec x \cdot \tan x$ |
| Pythagoras | $\sin^2 = 1-\cos^2$ | $\cos^2 = 1-\sin^2$ | $\sec^2 = 1+\tan^2$ | $\tan^2 = \sec^2 - 1$ |
| $u=$ | $\cos x$ | $\sin x$ | $\tan x$ | $\sec x$ |
| $du=$ | $-\sin x\,dx$ | $\cos x\,dx$ | $\sec^2 x\,dx$ | $\sec x \tan x\,dx$ |

ex1 $\int \sec^3 x \cdot \tan^3 x\,dx = \int \sec^2 x \cdot \tan^2 x \cdot \sec x \cdot \tan x\,dx$

$u = \sec x$

$du = \sec x \tan x\,dx$

$= \int \sec^2 x \cdot (\sec^2 x - 1) \cdot \sec x \cdot \tan x\,dx$

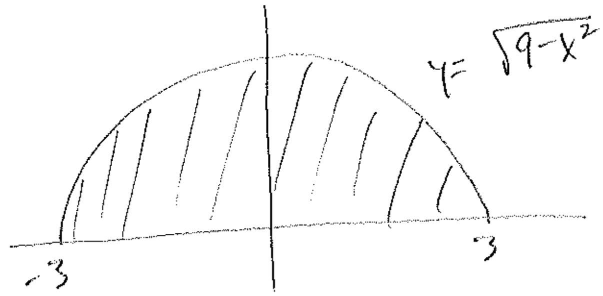
$= \int u^2(u^2-1)\,du = \int u^4 - u^2\,du = \frac{u^5}{5} - \frac{u^3}{3} + C$

$= \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$

Megan 2/15 P.2

§ 7.3 Trig Sub

Motivating example: find area of the circle $x^2+y^2=9$




$$\text{Area} = 2 \cdot \int_{-3}^{3} \sqrt{9-x^2}\, dx$$

Problem: we have no method for integrating $y=\sqrt{9-x^2}$.

| expression | substitution | Pythagorus |
|---|---|---|
| $\sqrt{a^2-x^2}$ | $x=a\sin\theta$ | $a^2-a^2\sin^2\theta=a^2\cos^2\theta$ |
| $\sqrt{a^2+x^2}$ | $x=a\tan\theta$ | $a^2+a^2\tan^2\theta=a^2\sec^2\theta$ |
| $\sqrt{x^2-a^2}$ | $x=a\sec\theta$ | $a^2\sec^2\theta-a^2=a^2\tan^2\theta$ |

ex 1 $\displaystyle\int \frac{x^3}{\sqrt{x^2-1}}\,dx$ $\qquad x = \sec\theta \quad dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-1} = \sqrt{\sec^2\theta - 1} = \sqrt{\tan^2\theta} = \tan\theta$

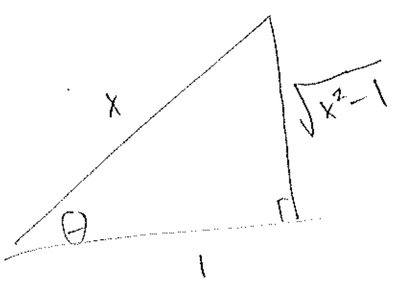
$$= \int \frac{\sec^3\theta}{\tan\theta}\cdot\sec\theta\cdot\tan\theta\,d\theta = \int \sec^4\theta\,d\theta = \int \sec^2\theta\cdot\sec^2\theta\,d\theta$$

$u = \tan\theta \qquad du = \sec^2\theta\,d\theta$

$$= \int (1+\tan^2\theta)\cdot\sec^2\theta\,d\theta$$

$$= \int 1+u^2\,du = u + \frac{u^3}{3} = \tan\theta + \frac{1}{3}\tan^3\theta$$

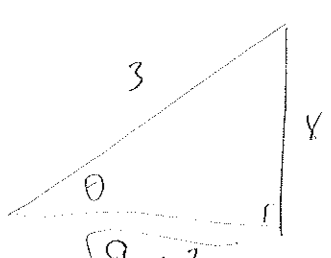
$$= \sqrt{x^2-1} + \frac{1}{3}(x^2-1)^{3/2} + C$$




ex 1 $\displaystyle\int \frac{x-1}{\sqrt{9-x^2}}\,dx$ $\qquad x = 3\sin\theta \quad dx = 3\cos\theta\,d\theta$

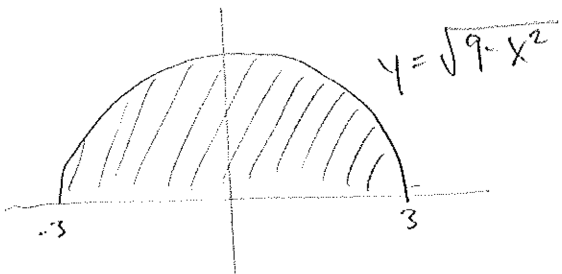
$$= \int \frac{3\sin\theta - 1}{3\cos\theta}\cdot 3\cos\theta\,d\theta = \int 3\sin\theta - 1\,d\theta = -3\cos\theta - \theta$$

$$= -3\cdot\frac{\sqrt{9-x^2}}{3} - \arcsin\left(\frac{x}{3}\right) + C$$




$$= -\sqrt{9-x^2} - \arcsin\left(\frac{x}{3}\right) + C$$

Motivating example: find area of circle $x^2+y^2=9$




$$\text{Area} = 2\cdot\int_{-3}^{3}\sqrt{9-x^2}\,dx$$

Problem: we have no method for integrating $y=\sqrt{9-x^2}$

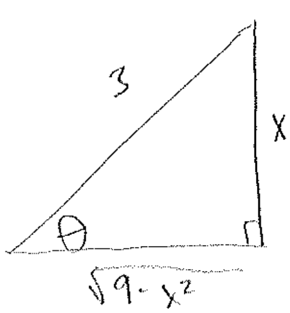
(u-sub does NOT work).

$x = 3\sin\theta \qquad dx = 3\cos d\theta$

$$\sqrt{9-x^2} = \sqrt{9-9\sin^2\theta} = \sqrt{9(1-\sin^2\theta)} = \sqrt{9\cos^2\theta} = 3\cos\theta$$

$$\text{Area} = 2\cdot\int_{-3}^{3}\sqrt{9-x^2}\,dx = 2\int_{x=-3}^{x=3} 3\cos\theta\cdot 3\cos\theta\,d\theta = 18\int_{x=-3}^{x=3}\cos^2\theta\,d\theta$$

$$= 18\cdot\frac{1}{2}\left(\theta+\sin\theta\cdot\cos\theta\right)\Big|_{x=-3}^{x=3} = 9\cdot\left(\arcsin\left(\frac{x}{3}\right)+\frac{x}{3}\cdot\frac{\sqrt{9-x^2}}{3}\right)\Big|_{-3}^{3}$$




$\sin\theta = \frac{x}{3}$

hence $\theta = \arcsin\left(\frac{x}{3}\right)$

and $\cos\theta = \frac{\sqrt{9-x^2}}{3}$

$= 9\cdot\arcsin(1) - 9\cdot\arcsin(-1)$

$= 9\cdot\frac{\pi}{2} - 9\left(\frac{-\pi}{2}\right)$

$= 9\pi$

soh cah toa