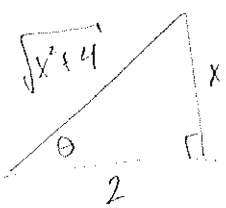
§7.3 Trig Sub

$$\int \frac{x}{\sqrt{x^2+4}}\,dx \qquad x = 2\tan\theta \qquad dx = 2\sec^2\theta\,d\theta$$

$$\sqrt{x^2+4} = \sqrt{4\sec^2\theta} = 2\sec\theta$$

$$= \int \frac{2\tan\theta}{2\sec\theta}\cdot 2\sec^2\theta\,d\theta = 2\int \tan\theta\sec\theta\,d\theta = 2\sec\theta + C$$

$$= \frac{2\cdot\sqrt{x^2+4}}{2} + C = \sqrt{x^2+4} + C$$



However, there is an easier method: $u = x^2+4 \quad du = 2x\,dx$

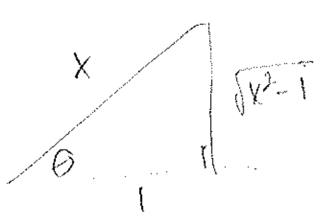
$$\int \frac{x}{\sqrt{x^2+4}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du = \sqrt{u} + C = \sqrt{x^2+4} + C$$

ex] $\displaystyle\int \frac{1}{\sqrt{x^2-1}}\,dx \qquad x = \sec\theta \quad dx = \sec\theta\tan\theta\,d\theta$

$$\sqrt{x^2-1} = \sqrt{\tan^2\theta} = \tan\theta$$

$$= \int \frac{1}{\tan\theta}\cdot\sec\theta\tan\theta\,d\theta = \int \sec\theta = \ln\left|\sec\theta + \tan\theta\right| + C$$

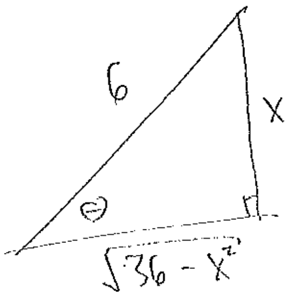
$$= \ln\left|x + \sqrt{x^2-1}\right| + C$$



Ex] $\displaystyle\int_0^3 \frac{x}{\sqrt{36-x^2}}\,dx$ $\qquad x = 6\sin\theta \qquad dx = 6\cos\theta\, d\theta$

$\sqrt{36-x^2} = \sqrt{36\cos^2\theta} = 6\cos\theta$

$$= \int_{x=0}^{x=3} \frac{6\sin\theta}{6\cos\theta}\cdot 6\cos\theta\, d\theta = -6\cos\theta \Big|_{x=0}^{x=3}$$



$$= -\sqrt{36-x^2}\Big|_{x=0}^{x=3} = -\sqrt{27} + 6$$

Ex] $\displaystyle\int \frac{x^5}{\sqrt{x^2+2}}\,dx$ $\qquad x = \sqrt{2}\tan\theta \qquad dx = \sqrt{2}\sec^2\theta\, d\theta$

$\sqrt{x^2+2} = \sqrt{2\sec^2\theta} = \sqrt{2}\cdot\sec\theta$

$$= \int \frac{\sqrt{2}^5\cdot\tan^5\theta}{\sqrt{2}\sec\theta}\cdot\sqrt{2}\sec^2\theta\, d\theta = 4\sqrt{2}\int \tan^5\theta\cdot\sec\theta\, d\theta$$

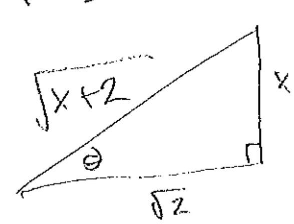
$u = \sec\theta \qquad du = \tan\theta\sec\theta\, d\theta$

$$= 4\sqrt{2}\int \tan^4\theta\cdot\sec\theta\cdot\tan\theta\, d\theta$$

$$= 4\sqrt{2}\int (\sec^2\theta - 1)^2\sec\theta\tan\theta\, d\theta = 4\sqrt{2}\int (u^2-1)^2\, du$$

$$= 4\sqrt{2}\int u^4 - 2u^2 + 1\, du = 4\sqrt{2}\left(\frac{u^5}{5} - \frac{2}{3}u^3 + u\right) = 4\sqrt{2}\left(\frac{\sec^5\theta}{5} - \frac{2}{3}\sec^3\theta + \sec\theta\right)$$

$$\sqrt{2}\left(\frac{1}{5}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right)^5 - \frac{2}{3}\left(\frac{\sqrt{x+2}}{\sqrt{2}}\right)^3 + \frac{\sqrt{x+2}}{\sqrt{2}}\right) + C$$



ex1 $\displaystyle\int \frac{1}{4+x^2}\,dx$ $\quad x = 2\tan\theta \quad dx = 2\sec^2\theta\, d\theta$

$4+x^2 = 4+4\tan^2\theta = 4\sec^2\theta$

$$= \int \frac{1}{4\sec^2\theta}\cdot 2\sec^2\theta\, d\theta = \int \frac{1}{2}\,d\theta = \frac{\theta}{2} = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$

ex1 $\displaystyle\int \frac{\sqrt{9-4x^2}}{x^2}\,dx$ $\quad x = \frac{3}{2}\sin\theta \quad dx = \frac{3}{2}\cos\theta\, d\theta$

$\sqrt{9-4x^2} = \sqrt{9-9\sin^2\theta} = 3\cos\theta$

$$= \int \frac{3\cos\theta}{\frac{9}{4}\sin^2\theta}\cdot \frac{3}{2}\cos\theta\, d\theta = 2\int \frac{\cos^2\theta}{\sin^2\theta}\,d\theta = 2\int \cot^2\theta\, d\theta$$

$$= 2\left(-\cot\theta - \theta\right) + C = 2\left(-\frac{\sqrt{9-4x^2}}{2x} - \arcsin\left(\frac{2x}{3}\right)\right) + C$$