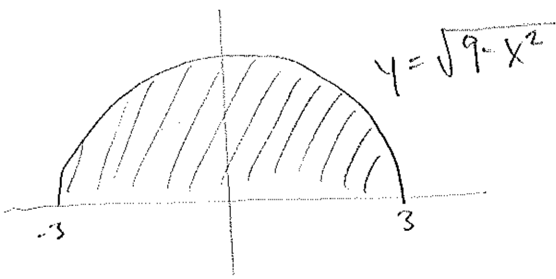
## §7.3 Trig Sub

Motivating example: find area of circle $x^2+y^2=9$




$$\text{Area} = 2\cdot\int_{-3}^{3}\sqrt{9-x^2}\,dx$$

Problem: we have no method for integrating $y=\sqrt{9-x^2}$ (u-sub does NOT work).

$x=3\sin\theta \qquad dx = 3\cos d\theta$

$$\sqrt{9-x^2} = \sqrt{9-9\sin^2\theta} = \sqrt{9(1-\sin^2\theta)} = \sqrt{9\cos^2\theta} = 3\cos\theta$$

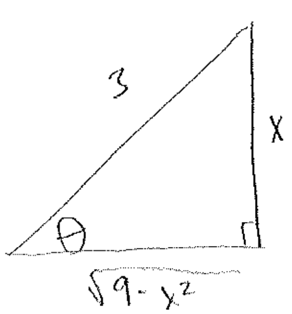
$$\text{Area} = 2\cdot\int_{-3}^{3}\sqrt{9-x^2}\,dx = 2\int_{x=-3}^{x=3} 3\cos\theta\cdot 3\cos\theta\,d\theta = 18\int_{x=-3}^{x=3}\cos^2\theta\,d\theta$$

$$= 18\cdot\tfrac{1}{2}\left(\theta+\sin\theta\cdot\cos\theta\right)\Big|_{x=-3}^{x=3} = 9\cdot\left(\arcsin\left(\tfrac{x}{3}\right)+\tfrac{x}{3}\cdot\tfrac{\sqrt{9-x^2}}{3}\right)\Big|_{-3}^{3}$$

$$= 9\cdot\arcsin(1) - 9\cdot\arcsin(-1)$$

$$= 9\cdot\tfrac{\pi}{2} - 9\left(\tfrac{-\pi}{2}\right)$$

$$= 9\pi$$




$\sin\theta = \frac{x}{3}$

hence $\theta = \arcsin\left(\frac{x}{3}\right)$

and $\cos\theta = \dfrac{\sqrt{9-x^2}}{3}$

soh cah toa

<u>ex 1</u> $\displaystyle\int \frac{x^3}{\sqrt{x^2-1}}\,dx$ $\qquad x = \sec\theta \qquad dx = \sec\theta\tan\theta\,d\theta$

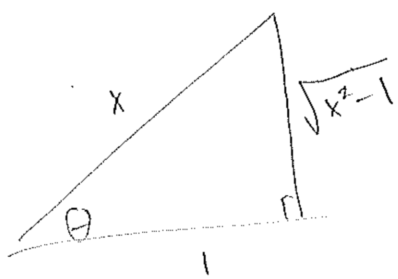
$\sqrt{x^2-1} = \sqrt{\sec^2\theta - 1} = \sqrt{\tan^2\theta} = \tan\theta$

$$= \int \frac{\sec^3\theta}{\tan\theta}\cdot\sec\theta\cdot\tan\theta\,d\theta = \int \sec^4\theta\,d\theta = \int \sec^2\theta\cdot\sec^2\theta\,d\theta$$

$$= \int (1+\tan^2\theta)\cdot\sec^2\theta\,d\theta \qquad u = \tan\theta \qquad du = \sec^2\theta\,d\theta$$

$$= \int 1 + u^2\,du = u + \frac{u^3}{3} = \tan\theta + \frac{1}{3}\tan^3\theta$$

$$= \sqrt{x^2-1} + \frac{1}{3}(x^2-1)^{3/2} + C$$

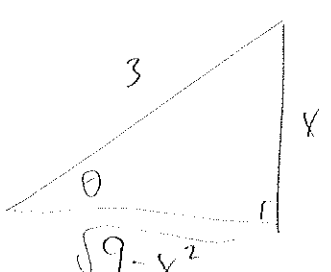



<u>ex 1</u> $\displaystyle\int \frac{x-1}{\sqrt{9-x^2}}\,dx$ $\qquad x = 3\sin\theta \qquad dx = 3\cos\theta\,d\theta$

$$= \int \frac{3\sin\theta - 1}{3\cos\theta}\cdot 3\cos\theta\,d\theta = \int 3\sin\theta - 1\,d\theta = -3\cos\theta - \theta$$

$$= -3\cdot\frac{\sqrt{9-x^2}}{3} - \arcsin\left(\frac{x}{3}\right) + C$$

$$= -\sqrt{9-x^2} - \arcsin\left(\frac{x}{3}\right) + C$$



# Table of Trig Sub

| expression | substitution | Pythagorus |
|---|---|---|
| $\sqrt{a^2 - x^2}$ | $x = a\sin\theta$ | $a^2 - a^2\sin^2\theta = a^2\cos^2\theta$ |
| $\sqrt{a^2 + x^2}$ | $x = a\tan\theta$ | $a^2 + a^2\tan^2\theta = a^2\sec^2\theta$ |
| $\sqrt{x^2 - a^2}$ | $x = a\sec\theta$ | $a^2\sec^2\theta - a^2 = a^2\tan^2\theta$ |