last time: $\int \sin^n x \cdot \cos^n x \, dx$ $\qquad$ $\int \tan^n x \cdot \sec^n x \, dx$

$1 = \cos^2 x + \sin^2 x$ $\qquad$ $\sec^2 x = 1 + \tan^2 x$

ex) $\int \cos^5 x \cdot \sin^2 x \, dx = \int \cos^4 x \cdot \sin^2 x \cdot \cos x \, dx$

$= \int (1-\sin^2 x)^2 \sin^2 x \cos x \, dx$ $\qquad$ $u = \sin x \quad du = \cos x \, dx$

$= \int (1-u^2)^2 \cdot u^2 \, du = \int (1 - 2u^2 + u^4) u^2 \, du$

$= \int u^2 - 2u^4 + u^6 \, du = \frac{u^3}{3} - \frac{2}{5}u^5 + \frac{1}{7}u^7 + C$

$= \frac{\sin^3 x}{3} - \frac{2}{5}\sin^5 x + \frac{1}{7}\sin^7 x + C$

ex) $\int \tan^6 x \sec^4 x \, dx$ $\qquad$ eventually want u-sub

$u = \tan x \quad du = \sec^2 x \, dx$

$\int \tan^6 x \sec^2 x \cdot \sec^2 x \, dx = \int \tan^6 x (1 + \tan^2 x) \sec^2 x \, dx$

$= \int u^6 (1 + u^2) \, du = \int u^6 + u^8 \, du = \frac{u^7}{7} + \frac{u^9}{9} + C$

$= \frac{1}{7}\tan^7 x + \frac{1}{9}\tan^9 x + C$

Note: the technique from the last example works when the power of secant is even.

ex) $\int \tan^5 x \cdot \sec^3 x \, dx$

- odd power of sec, can NOT save $\sec^2$ and rewrite remaining powers of sec via $\sec^2 = 1 + \tan^2$
- use the u-sub $u = \sec x$ $du = \sec x \cdot \tan x \, dx$

$\int \tan^4 x \cdot \sec^2 x \cdot \sec x \tan x \, dx$

$= \int (\sec^2 x - 1)^2 \cdot \sec^2 x \cdot \sec x \tan x \, dx$

$= \int (u^2 - 1)^2 u^2 \, du = \int (u^4 - 2u^2 + 1) u^2 \, du$

$= \int u^6 - 2u^4 + u^2 \, du = \frac{u^7}{7} - \frac{2}{5} u^5 + \frac{u^3}{3} + C$

$= \frac{1}{7} \sec^7 x - \frac{2}{5} \sec^5 x + \frac{1}{3} \sec^3 x + C$

In general:

	odd sin	odd cos	even sec	odd tan
same	$\sin x$	$\cos x$	$\sec^2 x$	$\sec x \tan x$
Pythagorous	$\sin^2 = 1 - \cos^2$	$\cos^2 = 1 - \sin^2$	$\sec^2 = 1 + \tan^2$	$\tan^2 = \sec^2 - 1$
$u =$	$\cos x$	$\sin x$	$\tan x$	$\sec x$
$du =$	$-\sin x\,dx$	$\cos x\,dx$	$\sec^2 x\,dx$	$\sec x \tan x\,dx$

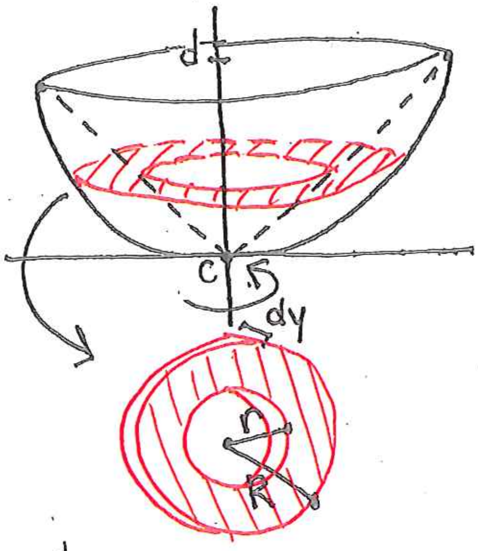
The Volume Cheat Sheet

* Note that these are the situations when rotating around axes. Modifications may be required to rotate about other lines.

Disk/Washer | Cylindrical Shells

Rotating about y-axis

Disk/Washer

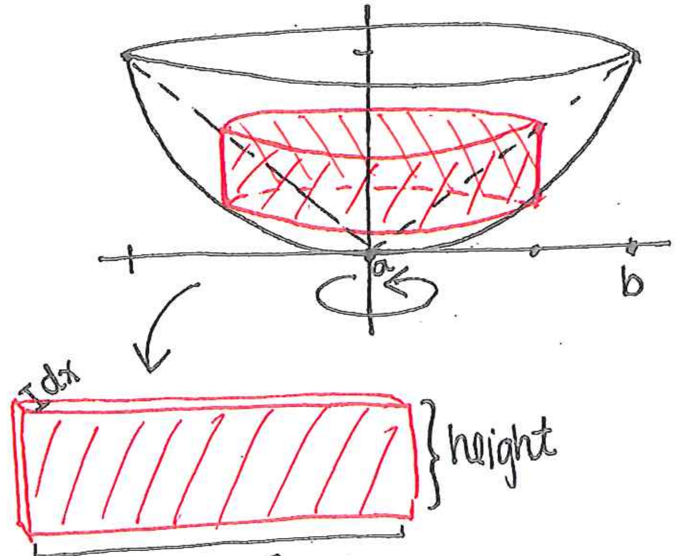


$$Vol = \int_c^d A(y)\,dy$$

$$A(y) = \pi R^2 - \pi r^2$$

this integral in terms of y

Cylindrical Shells



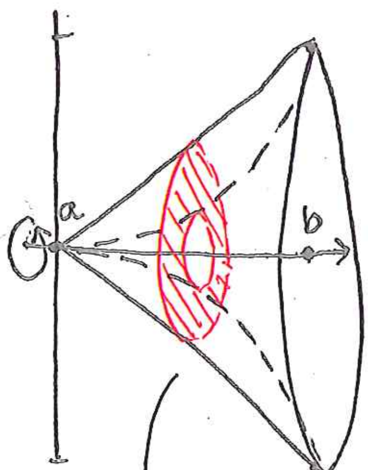
$$Vol = \int_a^b 2\pi x\, f(x)\,dx$$

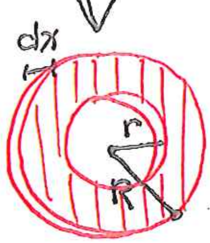
$f(x)$ = height of a cyl. shell w/ radius x

this integral in terms of x

Rotating about x-axis

Disk/Washer



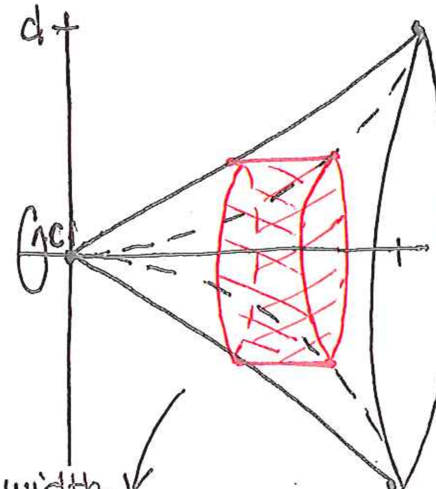


$$Vol = \int_a^b A(x)\,dx$$

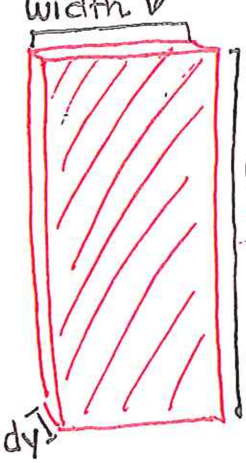
$$A(x) = \pi R^2 - \pi r^2$$

this integral in terms of x

Cylindrical Shells



$$Vol = \int_c^d 2\pi y\, f(y)\,dy$$



Circumference = $2\pi y$

$f(y)$ = width of a cyl. shell w/ radius y.

this integral in terms of y

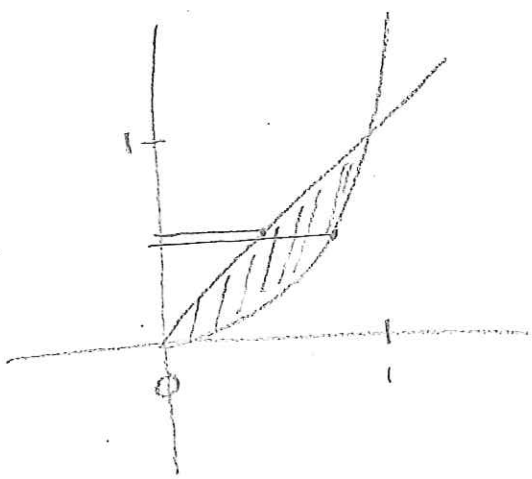
Quiz 3: Volume

February 6, 2013

Name: Key Section: Adelstein

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. Answer both questions (second one is on the back).

1. Use the disk or washer method to find the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = x^2$ about the y-axis.



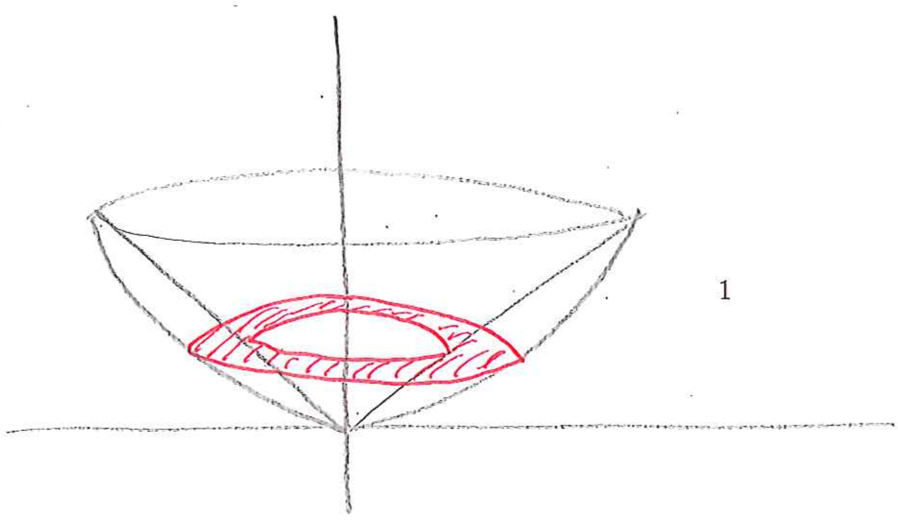
inner : y

outer : $\sqrt{y}$

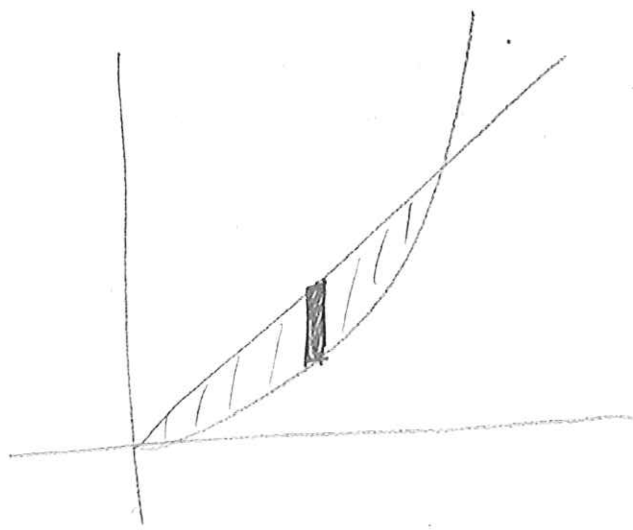
$A(y) = \pi(y - y^2)$

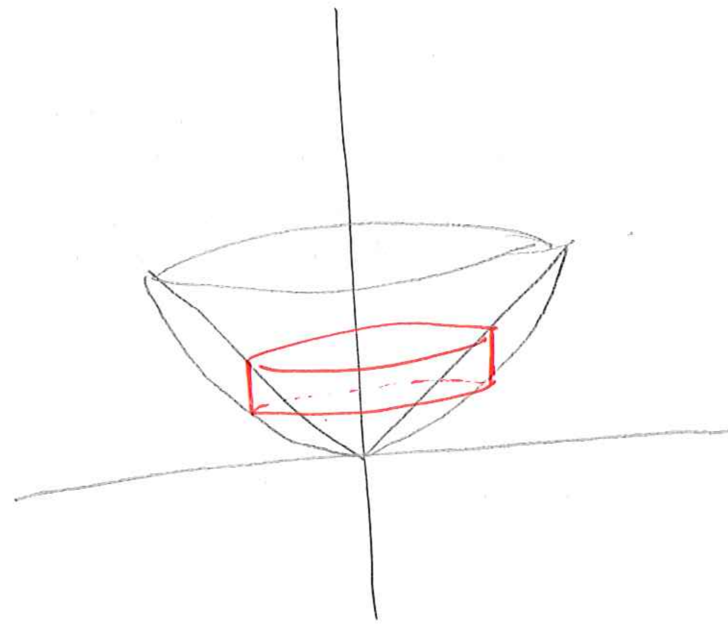
$$\text{Vol} = \int_0^1 \pi(y-y^2)\,dy = \pi\left(\frac{y^2}{2} - \frac{y^3}{3}\right)\Bigg|_0^1$$

$$= \pi\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{\pi}{6}$$



2. Use the cylindrical shells method to find the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = x^2$ about the y-axis.





$f(x) =$ height of washer $= x - x^2$

$$\text{Vol} = \int_0^1 2\pi x(x - x^2)\,dx = 2\pi \int_0^1 x^2 - x^3\,dx$$

$$= 2\pi \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Bigg|_0^1 = 2\pi\left(\frac{1}{3} - \frac{1}{4}\right)$$

$$= 2\pi\left(\frac{4}{12} - \frac{3}{12}\right) = 2\pi\left(\frac{1}{12}\right) = \frac{\pi}{6}$$