## Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

ex) $\int x \sin x\, dx$ $\quad u = x \quad dv = \sin x\, dx$
$\quad du = dx \quad v = -\cos x$

$$= \int u\,dv = uv - \int v\,du = -x\cos x - \int (-\cos x)\,dx$$
$$= -x\cos x + \sin x + C$$

Q: what makes a good choice for $u$?

A: part of the integrand that becomes simpler when differentiated.

$u = \sin x \quad dv = x\,dx$
$du = \cos x\,dx \quad v = \frac{x^2}{2}$

$$\int x \sin x\,dx = \sin x \cdot \frac{x^2}{2} - \int \frac{x^2}{2}\cdot \cos x\,dx$$

this is True, but does not make the integral simpler.

ex| $\int x^2 e^x \, dx$ $\qquad u = x^2 \quad dv = e^x \, dx$

$\qquad du = 2x \, dx \quad v = e^x$

$= x^2 e^x - \int e^x \cdot 2x \, dx$ $\qquad u = 2x \quad dv = e^x \, dx$

$\qquad du = 2 dx \quad v = e^x$

$= x^2 e^x - \left(2xe^x - \int 2e^x \, dx\right)$

$= x^2 e^x - 2xe^x + 2e^x + C$

ex| $\int \ln x \, dx$ $\qquad u = \ln x \quad dv = dx$

$\qquad du = \frac{1}{x} dx \quad v = x$

$= x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - x + C$

classroom example here.

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ex| $\int_0^1 \arctan x \, dx$ $\qquad u = \arctan x \quad dv = dx$

$\qquad du = \frac{1}{1+x^2} dx \quad v = x$

$= x \cdot \arctan x - \int \frac{x}{1+x^2} dx$ $\qquad t = 1 + x^2 \quad dt = 2x \, dx$

$\qquad \frac{1}{2} dt = x \, dx$

$= x \cdot \arctan x - \frac{1}{2} \int \frac{1}{t} dt = x \cdot \arctan x - \frac{1}{2} \ln t$

$= x \arctan x - \frac{1}{2} \ln(1+x^2) \Big|_0^1 = \arctan(1) - \frac{1}{2}\ln(2) - \left(\arctan(0) - \frac{1}{2}\ln(1)\right)$

$= \pi/4 - \frac{1}{2}\ln(2)$

ex 1 $\int x^2 \ln x \, dx$

$u = \ln x \qquad dv = x^2\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^3}{3}$

$$= \frac{x^3}{3}\cdot \ln x - \int \frac{x^2}{3}\,dx = \frac{x^3}{3}\cdot \ln x - \frac{x^3}{9} + C$$

what if you choose $u = x^2 \qquad dv = \ln x\,dx$

$du = 2x\,dx \qquad v = x\ln x - x$

$$= x^2(x\ln x - x) - \int 2x^2 \ln x - 2x^2\,dx$$

this did NOT help