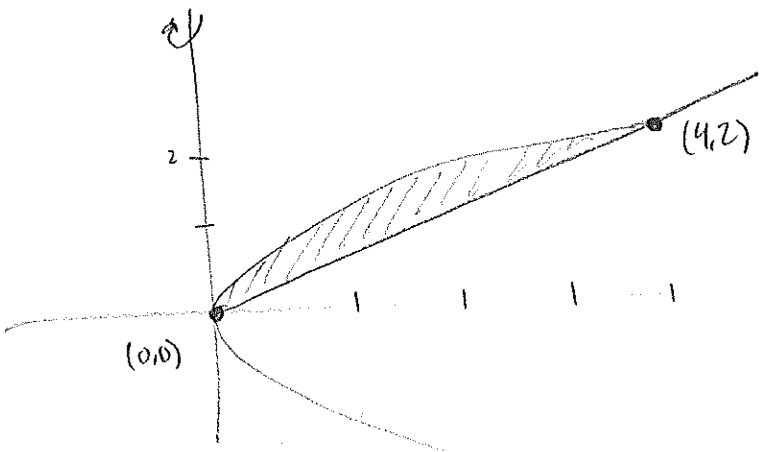
Volumes :

ex 1 find volume obtained by rotating the region bounded by

$y^2 = x \qquad x = 2y \qquad$ about y-axis.

$y^2 = 2y$

$y = 2,\ y = 0$



which method?

inner radius : y^2 outer radius : $2y$

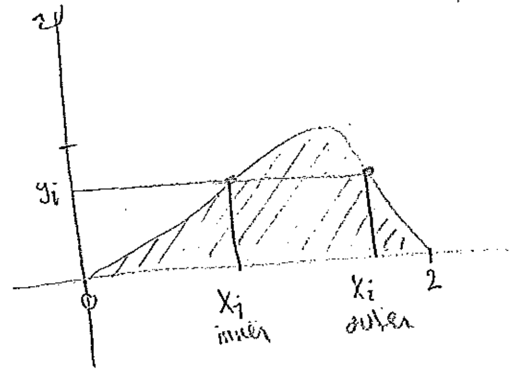
$$A(y) = \pi\left(4y^2 - y^4\right)$$

$$Vol = \int_0^2 \pi\left(4y^2 - y^4\right)dy = \pi\left(\frac{4}{3}y^3 - \frac{y^5}{5}\right)\Big|_0^2 = \pi\left(\frac{32}{3} - \frac{32}{5}\right) = \frac{64}{15}\pi$$

6.3 Volume by Cylindrical Shells

Ex] $y = 2x^2 - x^3$ and $y = 0$

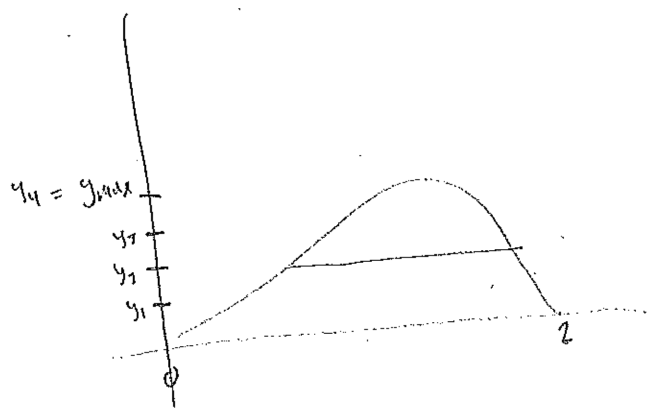
rotate about the y-axis



Q: disk or washer? A: neither

Washer: $\int_0^{y_{max}} A(y)\,dy$ where $A(y_i) = \pi\left(\underset{\text{outer}}{x_j^2} - \underset{\text{inner}}{x_i^2}\right)$

The washer method becomes very hard for this problem....



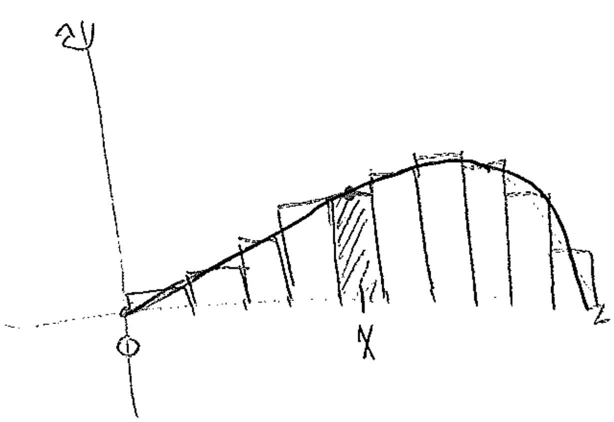
$$\Delta y = \frac{y_{max} - 0}{n}$$

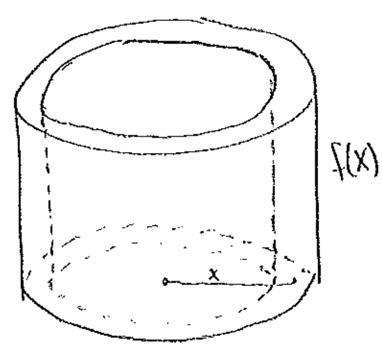
$$Vol = \lim_{n \to \infty} \sum_{i=1}^{n} A(y_i)\,\Delta y$$

area of i^{th} washer

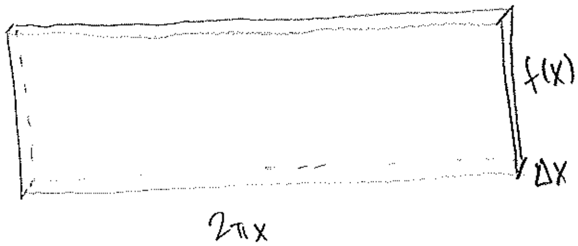
Cylindrical Shells:

spin rectangle about y-axis yielding a cylindrical shell





now just lay the cylindrical shell flat

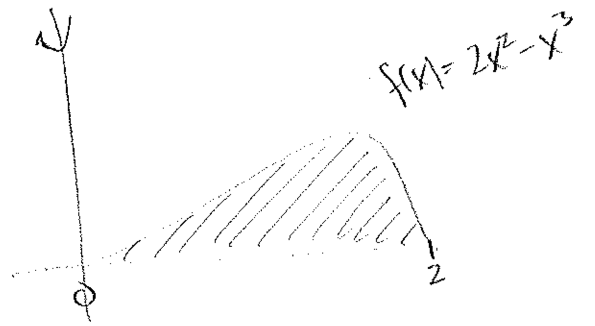


$$Vol = 2\pi x \cdot f(x) \cdot \Delta x$$

$$\boxed{Vol = \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi x_i \cdot f(x_i) \cdot \Delta x = \int_a^b 2\pi x \cdot f(x)\, dx}$$

The volume of the solid obtained by rotating about the y-axis the region under the curve $y=f(x)$ from a to b.

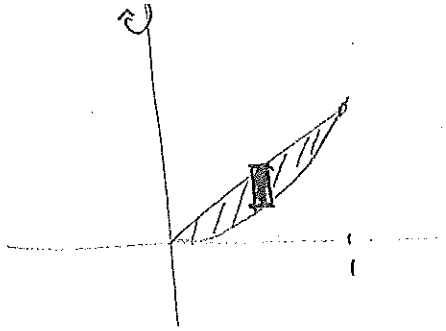
ex)



$$Vol = \int_0^2 2\pi x \cdot (2x^2 - x^3)\, dx = 2\pi \int_0^2 2x^3 - x^4\, dx = 2\pi \left(\frac{x^4}{2} - \frac{x^5}{5} \right) \Big|_0^2$$

$$= 2\pi \left(8 - \frac{32}{5} \right) = \frac{16}{5}\pi$$

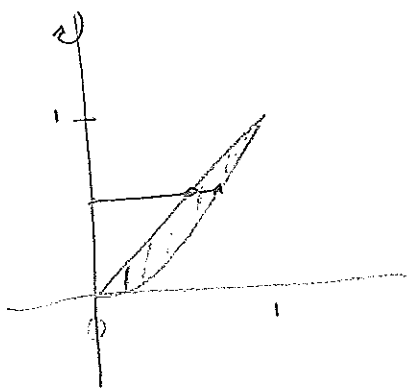
ex] find volume of solid obtained by rotating about the y-axis the region between $y=x$ and $y=x^2$



$$Vol = \int_0^1 2\pi x (x - x^2)\, dx = 2\pi \int_0^1 x^2 - x^3\, dx$$

$$= 2\pi \left(\frac{x^3}{3} - \frac{x^4}{4} \right) \Big|_0^1 = 2\pi \left(\frac{1}{3} - \frac{1}{4} \right) = \frac{\pi}{6}$$

Now use washer method: $y=x$ and $x = \sqrt{y}$



$$A(y) = \pi (\sqrt{y})^2 - \pi y^2 = \pi (y - y^2)$$

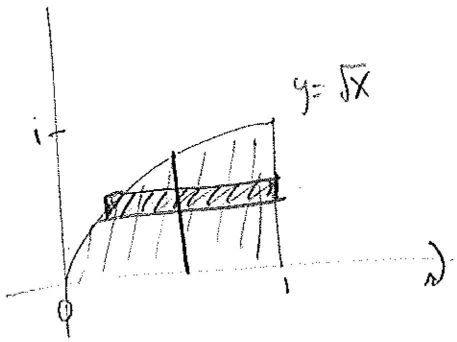
$$Vol = \int_0^1 \pi (y - y^2)\, dy = \pi \left(\frac{y^2}{2} - \frac{y^3}{3} \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{\pi}{6}$$

Recap:

	disk / washer	cylindrical shells
y-axis	$Vol = \int_c^d A(y)\, dy \quad c \le y \le d$ $A(y)$ = area of cross-section $A(y) = \pi (r_{out}^2 - r_{in}^2)$	$Vol = \int_a^b 2\pi x \cdot f(x)\, dx$ $a \le x \le b$ $f(x)$ = height of shell
x-axis	$Vol = \int_a^b A(x)\, dx \quad a \le x \le b$ $A(x)$ = area of cross-section $A(x) = \pi (r_{out}^2 - r_{in}^2)$	$Vol = \int_c^d 2\pi y \cdot f(y)\, dy$ $c \le y \le d$ $f(y)$ = width of shell

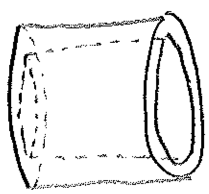
ex. find volume rotating about x-axis region under $y=\sqrt{x}$ from 0 to 1

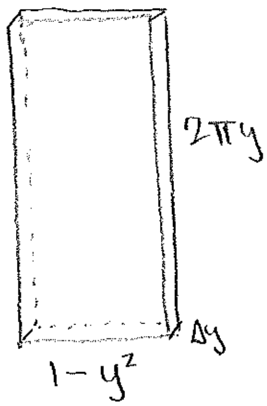


disks: $A(x) = \pi(\sqrt{x})^2 = \pi x$

$$\text{Vol} = \int_0^1 \pi x \, dx = \pi \frac{x^2}{2}\Big|_0^1 = \frac{\pi}{2}$$

Shells:





$$\text{Vol} = \int_0^1 2\pi y\,(1-y^2)\,dy = 2\pi\int_0^1 y - y^3\,dy = 2\pi\left(\frac{y^2}{2} - \frac{y^4}{4}\right)\Big|_0^1 = 2\pi\left(\frac{1}{2}-\frac{1}{4}\right) = \frac{\pi}{2}$$