Quiz 8: Improper Integrals

February 29, 2012

Name: Solutions Section:

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. This quiz has two sides.

1. Find $\int_0^3 \frac{1}{x^3}\,dx$.

$$= \lim_{t\to 0^+} \int_t^3 \frac{1}{x^3}\,dx = \lim_{t\to 0^+} \left(-\frac{1}{2x^2}\right)\Big|_t^3$$

$$= \lim_{t\to 0^+} \left(-\frac{1}{2(3)^2} + \frac{1}{2t^2}\right) = -\frac{1}{18} + \infty = \infty$$

so the integral <u>diverges</u>.

2. Find $\int_0^\infty xe^{-x}\,dx$. $= \lim_{t\to\infty} \int_0^t xe^{-x}\,dx$

IBP: $u = x \quad dv = e^{-x}dx$
$du = dx \quad v = -e^{-x}$

$$= \lim_{t\to\infty} \left(-xe^{-x} + \int e^{-x}dx\right)\Big|_0^t = \lim_{t\to\infty} \left(-xe^{-x} - e^{-x}\right)\Big|_0^t$$

$$= \lim_{t\to\infty} \left(-t e^{-t} - e^{-t} + 0 + e^0\right)$$

$$= \underbrace{\lim_{t\to\infty}(-te^{-t})}_{-\infty\cdot 0} - \underbrace{\lim_{t\to\infty} e^{-t}}_{0} + 0 + 1$$

$-\infty \cdot 0$
indeterminate form.

$$-\lim_{t\to\infty} te^{-t} = -\lim_{t\to\infty} \frac{t}{e^t} \overset{L'H}{=} -\lim_{t\to\infty} \frac{1}{e^t} = 0$$

So $\int_0^\infty xe^{-x}dx = 0 - 0 + 0 + 1$
$= 1$

Extra Credit: Find a function $f(x)$ so that both $\int_0^1 f(x)\,dx$ and $\int_1^\infty f(x)\,dx$ diverge.

Try $f(x) = \frac{1}{x}$.