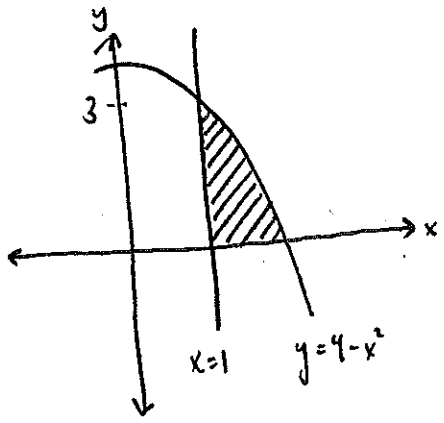
Quiz 5: Volumes!

February 8, 2012

Name: Solutions Section: _____

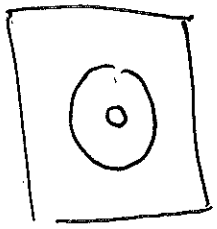
Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. Answer both questions (second one is on the back).

Consider the region bounded by the curves $y = 4 - x^2$, $x = 1$, and $y = 0$ as pictured to the right. Rotate this region about the y-axis to form a solid.



1. Compute the volume of this solid using disks/washers (slices).

$V = \int_a^b A(y)\,dy$. Cross-section: Washer.



$y = 4 - x^2 \Rightarrow x^2 = 4 - y \Rightarrow x = \sqrt{4-y^2}$.

$A(y) = \pi\left(\sqrt{4-y^2}\right)^2 - \pi(1)^2 = \pi(4-y) - \pi = \pi(3-y)$.

$$V = \int_0^3 \pi(3-y)\,dy = \pi\int_0^3 3-y\,dy = \pi\left[3y - \frac{1}{2}y^2\right]_0^3 = \pi\left[9 - \frac{9}{2}\right] = \boxed{\frac{9\pi}{2}}.$$

2. Compute the volume of this solid using cylindrical shells.

$$V = \int_c^d 2\pi x\, f(x)\, dx$$

To find the upper limit d, we solve $4 - x^2 = 0 \Rightarrow x = \pm 2$.

$$V = \int_1^2 2\pi x\,(4 - x^2)\,dx = 2\pi \int_1^2 (4x - x^3)\,dx = 2\pi \left[2x^2 - \frac{x^4}{4} \right]_1^2$$

$$= 2\pi \left[8 - 4 - 2 + \frac{1}{4} \right] = 2\pi \left[2 + \frac{1}{4} \right] = \boxed{\frac{9\pi}{2}}.$$

Bonus: Name an object in this classroom which is a solid of revolution.

Chalk stick.