Quiz 4: Area Between Curves

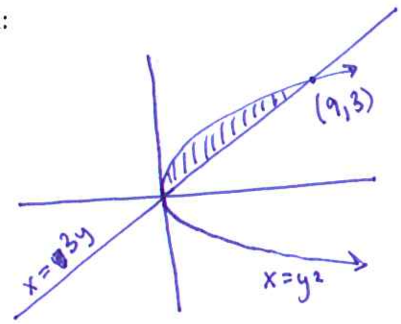
February 1, 2012

Name: Solutions. Section:____________________

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. This quiz has two sides.

1. Consider the region bounded by the curves $x = y^2$ and $x = 3y$. You will compute this area of this region in two different ways.

   (a) Sketch the region:



   (b) First, set up an integral with respect to y (so that your integral has dy in it) that will give you the area between the curves. Evaluate it.

$$A = \int_c^d f(y) - g(y)\,dy = \int_0^3 3y - y^2\,dy$$

$$\boxed{\begin{array}{c} y^2 = 3y \\ y^2 - 3y = 0 \\ y(y-3) = 0 \\ y = 0 \quad y = 3 \end{array}}$$

$$= \frac{3y^2}{2} - \frac{y^3}{3}\Big|_0^3$$

$$= \frac{3(3)^2}{2} - \frac{3^3}{3} - \left(\frac{3(0)^2}{2} - \frac{(0)^3}{3}\right)$$

$$= 3^3\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{3^3}{6} = \frac{9}{2}.$$

(c) Now find an integral with respect to x (so that your integral has dx in it) that will give you the area between the curves and evaluate it. Note that you should get the same answer as in part (b).

$$A = \int_a^b f(x) - g(x)\,dx$$

$$y = f(x) = \sqrt{x}$$
$$g(x) = \frac{x}{3}.$$

$$\boxed{\begin{aligned}\sqrt{x} &= \frac{x}{3}\\ x &= \frac{x^2}{9}\\ 9x &= x^2\\ x=0&,\ x=9\end{aligned}}$$

So,
$$A = \int_0^9 \sqrt{x} - \frac{x}{3}\,dx = \int_0^9 x^{1/2} - \frac{x}{3}\,dx = \frac{2x^{3/2}}{3} - \frac{x^2}{6}\Bigg|_0^9$$

$$= \frac{2(9)^{3/2}}{3} - \frac{9^2}{6} - \left(\frac{2\cdot 0^{3/2}}{3} - \frac{0^2}{6}\right) = \frac{2\cdot 3^3}{3} - \frac{9^2}{6} = 3^3\left(\frac{2}{3} - \frac{3}{6}\right) = \frac{27}{6} = \frac{9}{2}.$$

Extra Credit: Find the area of the region in the first quadrant bounded by curves $y = x^a$ and $y = x^b$ where $a > b > 0$.

$$\boxed{\begin{aligned}x^a &= x^b\\ x^a - x^b &= 0\\ x^b(x^{a-b} - 1) &= 0\\ x=0\quad & x=1\end{aligned}}$$

$$\int_0^1 x^b - x^a\,dx = \frac{x^{b+1}}{b+1} - \frac{x^{a+1}}{a+1}\Bigg|_0^1$$

$$= \frac{1^{b+1}}{b+1} - \frac{1^{a+1}}{a+1} - \left(\frac{0^{b+1}}{b+1} - \frac{0^{a+1}}{a+1}\right)$$

$$= \frac{1}{b+1} - \frac{1}{a+1}$$