

Quiz 3: Substitution

January 27, 2012

Name: _____ Section: _____

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. Answer both questions (second one is on the back).

1. Evaluate the indefinite integral

$$\int 2x^3(x^2 + 1)^{100} dx.$$

Let $u = x^2 + 1$. Then $\frac{du}{dx} = 2x$, so $dx = \frac{du}{2x}$.

$$\int 2x^3(x^2 + 1)^{100} dx = \int 2x^3 u^{100} \frac{du}{2x} = \int x^2 u^{100} du.$$

We can back substitute: $u = x^2 + 1 \Rightarrow x^2 = u - 1$.

$$\int x^2 u^{100} du = \int (u - 1) u^{100} du = \int (u^{101} - u^{100}) du = \frac{1}{102} u^{102} - \frac{1}{101} u^{101} + C$$

$$= \frac{1}{102} (x^2 + 1)^{102} - \frac{1}{101} (x^2 + 1)^{101} + C$$

2. Evaluate the definite integral

$$\int_{1/\pi}^{6/\pi} \frac{\cos(1/x)}{x^2} dx.$$

Let $u = \frac{1}{x}$. Then $\frac{du}{dx} = -\frac{1}{x^2}$, so $dx = -x^2 du$.

With respect to u , the limits of integration change: $\frac{1}{\pi} \rightsquigarrow \pi$ and $\frac{6}{\pi} \rightsquigarrow \frac{\pi}{6}$.

$$\int_{1/\pi}^{6/\pi} \frac{\cos(1/x)}{x^2} dx = \int_{\pi}^{\pi/6} \frac{\cos(u)}{x^2} (-x^2 du) = - \int_{\pi}^{\pi/6} \cos(u) du$$

$$= -\sin(u) \Big|_{\pi}^{\pi/6} = -\sin\left(\frac{\pi}{6}\right) + \sin(\pi) = \boxed{-\frac{1}{2}}$$

Extra Credit: What is $\int_{-20}^{20} \cos(x) \sin(\sqrt[3]{x}) dx$? Justify your answer.

The function $\cos(x) \sin(\sqrt[3]{x})$ is odd, so the integral is 0.