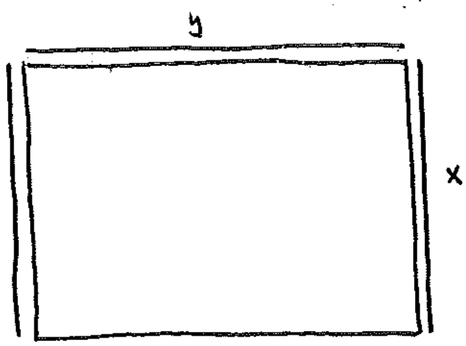
Quiz 1: Optimization and L'Hôpital's Rule

January 11, 2012

Name: ______________________ Section: ____________________

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. Answer both questions (second one is on the back).

1. You want to build a rectangular pen for your new puppy. To combat the fierce winter wind, you require that the sides facing the north, east, and west have two layers of fencing (with no space between them). The side facing the south only has one layer. (See picture.) If you have 600 feet of fencing, what dimensions will maximize the area of the pen (hence maximizing your puppy's happiness)?



fence constraint: $4x+3y=600$

area: $A=xy$

We want to maximize A. First we must eliminate all but one variable. We can use our fence constraint: $4x+3y=600 \Rightarrow 3y=600-4x$

$$\Rightarrow y=\frac{600-4x}{3}.$$

Now $A=x\left(\frac{600-4x}{3}\right)$ is a function of one variable.

Critical points: $A'(x)=(x)\left(-\frac{4}{3}\right)+(1)\left(\frac{600-4x}{3}\right)$ (product rule)

$$=-\frac{4}{3}x+\frac{600-4x}{3}=200-\frac{8}{3}x.$$

So $A'(x)=0 \Rightarrow \frac{8}{3}x=200 \Rightarrow x=75$. The dimensions are

Then $y=\frac{600-4x}{3}=\frac{600-4\cdot 75}{3}=100$. $\boxed{75' \times 100'}$

2. Use l'Hôpital's rule to find the limit.

$$\lim_{x\to 0} \frac{\sin(3x)}{x\cos(5x)}$$

$\lim_{x\to 0} \sin(3x) = 0$ $\qquad$ $\lim_{x\to 0} x\cos(5x) = 0.$

This is indeterminate form of type $\frac{0}{0}$.

$$\lim_{x\to 0} \frac{\sin(3x)}{x\cos(5x)} \overset{l'H}{=} \lim_{x\to 0} \frac{3\cos(3x)}{(1)\cos(5x) + (x)(-5\sin(5x))}$$ (product rule)

$$= \lim_{x\to 0} \frac{3\cos(3x)}{\cos(5x) - 5x\sin(5x)}$$

$$= \frac{3\cos(0)}{\cos(0) - 5\cdot 0\cdot 0}$$ plug in $x=0$

$$= \frac{3}{1-0} = \boxed{3.}$$