

NAME: _____

INSTRUCTOR: Gillaspy Gottschlich

MATH 2 PRACTICE FINAL EXAM

INSTRUCTIONS: This Practice Final is a good approximation to the length and difficulty of the final exam you will take on Saturday, March 12 at 3 PM in Kemeny 008. We will spend class time on Wednesday, March 9 reviewing for the final exam, including solving some of the problems from this practice exam. Please inform your instructor of any problems you would particularly like to see done in class.

Please show all of your work and justify all of your answers.

1. [10 points] Suppose that

$$f''(x) = \frac{4 + 20x^5}{x^2}, \quad f(-1) = -5, \quad f'(1) = 2.$$

Please find $f(x)$.

2. [10 points] Please find the length of the graph of the function

$$f(t) = \int_0^t \sqrt{2x^2 + 1} dx$$

on the interval $0 \leq t \leq 2$.

3. Let A be the area above the graph of $f(x) = \ln(x)$ over the interval $[0, 1]$.
- (a) [10 points] Is A finite? If so, how big is A ? *Please show all your work!*
- (b) [10 points] Let V be the volume formed by rotating A around the x axis. Is V finite? How big is V ? *Please show all your work!*

4. [8 points] Please evaluate the definite integral $\int_1^3 \frac{3x^2 - x + 4}{2x} dx$, or write “Divergent” if it diverges. *Please show all your work!*

5. [10 points] A patient received a 6-milligram injection of dye into the right atrium of her heart. Then the doctors measured the concentration $c(t)$ of dye, in milligrams per liter of blood, at 2-second time intervals. The data they collected are given below. Please use this data to estimate the patient's cardiac output.

t	0	2	4	6	8	10	12
$c(t)$	0	0.5	3	8	4	1.5	0

6. [8 points] Please evaluate the indefinite integral $\int \frac{\sin(x)}{1+\cos^2(x)} dx$.

7. [20 points] Let A be the region bounded by the lines $y = 5x$, $y = x^2 + 6$. Let V be the volume formed by rotating A around the line $x = 2$.
- (a) Please find the area of A .
 - (b) Please find V using the method of cylindrical shells.
 - (c) Please find V using the method of discs/washers.

8. [10 points] Please find the average value of the function $f(t) = t \sin(\pi t)$ on the interval $[1/2, 5]$.

9. [9 points] Which of the following integrals are improper? Why or why not?

(a)

$$\int_0^1 \frac{1}{\sqrt{2x^2 - x + 1}} dx$$

(b)

$$\int_e^\infty \frac{1}{x(\ln(x))^3} dx$$

(c)

$$\int_0^1 \sqrt{2x^2 - x + 1} dx$$

10. [10 points] In 1847, the population of Ireland was about 8 million, the relative birth rate was $b = 3.5\%$ of the population, and the relative death rate was $d = 1.9\%$ of the population.

(a) What was the relative growth rate k of the population of Ireland in 1847? Please justify your answer.

(b) Please write a formula $P(t)$ expressing the population of Ireland as a function of the time t , measured in years after 1847.

(c) According to your formula, when did the population of Ireland reach 10 million?

11. [10 points] Please use trigonometric substitution to evaluate the integral

$$\int \frac{x^3}{\sqrt{16 - x^2}} dx.$$

12. [10 points] Please evaluate the integral $\int_{-1}^4 4x^3 \ln(x) dx$, or write “Divergent” if it diverges. *Please show all your work!*