$$\int \cos^n x\,dx = \frac{1}{n}\sin x\cos^{n-1}x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$$

($n \geq 2$ is an integer)

$\int \cos^n x\,dx =$

$u = \cos^{n-1}x$ $\quad dv = \cos x\,dx$

$du = -(n-1)\cos^{n-2}x\sin x$ $\quad v = \sin x$

$\int u\,dv = uv - \int v\,du$

$= \sin x\cos^{n-1}x + \underset{\text{constant}}{-(n-1)}\int \cos^{n-2}x\sin^2 x\,dx$

Use "u" substitution because its integration by parts. Get du using the chain rule.

$(\sin^2 x = 1 - \cos^2 x)$

$\int \cos^n x\,dx = \sin x\cos^{n-1}x + (n-1)\int \cos^{n-2}x\,dx \overset{+}{-} (n-1)\int \cos^n x\,dx$

Now just like the $\int \sin^n x\,dx$, Solve by taking last term on the right side to the left and get.

$n\int \cos^n x\,dx = \sin x\cos^{n-1}x + (n-1)\int \cos^{n-2}x\,dx$

(divide by n) $= \underline{\frac{1}{n}\sin x\cos^{n-1}x + \frac{n-1}{n}\int \cos^{n-2}x\,dx}$

Solve $\int \cos^3 x\,dx$.

(1st way)

$\int \cos^{\overset{\text{"n"}}{3}} x\,dx = \frac{1}{3}\sin x\cos^2 x + \frac{2}{3}\int \cos x\,dx = \underline{\frac{1}{3}\sin x\cos^2 x + \frac{2}{3}\sin x + C}$

Check derivative:

$\frac{1}{3}\sin x \cdot 2\cos x(-\sin x) + \cos^2 x \cdot \frac{1}{3}\cos x + \frac{2}{3}\cos x$

$= -\frac{2}{3}\sin^2 x\cos x + \frac{1}{3}\cos^3 x + \frac{2}{3}\cos x \rightarrow -\frac{2}{3}(1-\cos^2 x)(\cos x) + \frac{1}{3}\cos^3 x + \frac{2}{3}\cos x \rightarrow$ ✓

$= -\frac{2}{3}(\cos x - \cos^3 x) + \frac{1}{3}\cos^3 x + \frac{2}{3}\cos x \rightarrow \cancel{-\frac{2}{3}\cos x} + \frac{2}{3}\cos^3 x + \frac{1}{3}\cos^3 x + \cancel{\frac{2}{3}\cos x} = \boxed{\cos^3 x}$

(2nd way) Power of cosine is odd so save one copy of $\cos x$, use $\cos^2 x = (1-\sin^2 x)$, let $u = \sin x$

$\int \cos^2 x\cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int (1-\sin^2 x)\cos x\,dx =$

$\int (1-u^2)\,du$

$= u - \frac{1}{3}u^3 + C$

$= \underline{\sin x - \frac{1}{3}\sin^3 x + C}$

Check derivative:

$\sin x - \frac{1}{3}(\sin x)^3 + C$ chain Rule

$\cos x - \frac{1}{3}\cdot 3(\sin x)^2 \cdot \cos x =$

$\cos x - \sin^2 x\cos x =$

$\cos x(1-\sin^2 x) =$

$(\cos^2 x)$ ✓

$\cos x(\cos^2 x) = \boxed{\cos^3 x}$