Math 2 Due: February 15, 2008

Written Problem 6

Unlike you, who couldn't be torn away from your calculus book all weekend, your two roommates attended parties on Super Bowl Sunday. They are both huge Patriots fans and decided no one party could handle both of their enthusiasms, so they went to separate parties. Interestingly enough both parties had exactly the same football shaped cake to serve to the guests. Since your roommates are both two young to drown their sorrows in adult beverages, they each ate the entire cake at their respective parties. Both of them have finally recovered enough from the shame to face the outside world and go to the gym to burn off those cake calories. They need your calculus expertise in order to figure out how much cake they actually ate. They have learned all about the cake's nutrition facts, so they just need you to figure out the volume of cake they each ate. At one party the cake was plain chocolate, so you roommate ate the entire thing. At the other party the cake had an ice cream center. Your roommate at that party couldn't eat that entire cake, lactose intolerant, so she settled for just the outer chocolate cake part. You have spoken to the bakery in town to learn some facts about the cakes. Both have a base that can be described as the region bounded by $y = \sin x$, $y = -\sin x$, and $0 \leqslant x \leqslant \pi$. The slices of cake, perpendicular to the x-axis, were perfect semi-circles. The ice cream center of the second cake was in the shape of half of a right circular cylinder. At the widest, or tallest, part of the cake, the ice cream extended from the center of the cake half way to the edge. Once the ice cream reached edge the cylinder was cut off, leaving the pointy ends of the cake to be all chocolate.

Find the volume of cake each roommate consumed. Write out your solution **neatly** and **carefully**. Use **complete sentences** to describe **all** the steps you took to arrive at your solution. You should be able to find the volume of the ice cream filled cake in two ways. Make sure the two answers agree.

You may find the following antiderivative very helpful:

$$\int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$