WRITTEN PROBLEM #3

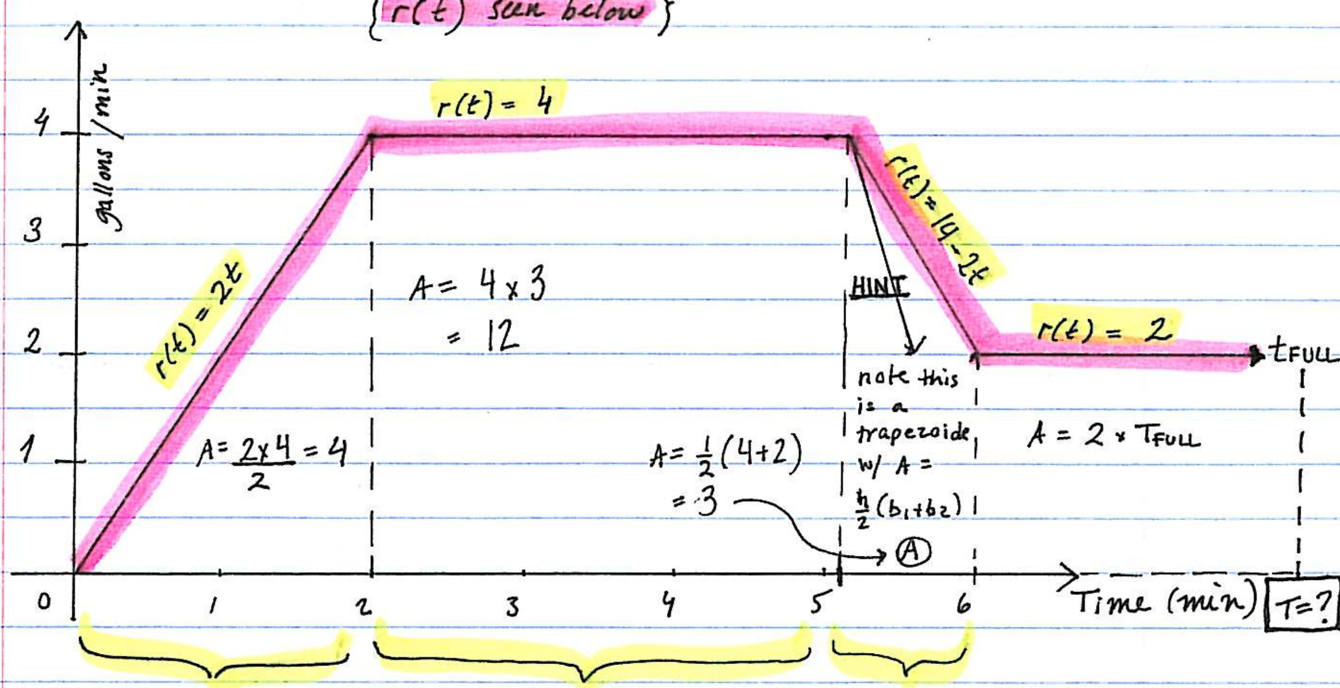
1. Draw a carefully labeled graph of this function.
See below:

$$r(t) = \begin{cases} 2t & \text{if } 0 \le t < 2 \\ 4 & \text{if } 2 \le t < 5 \\ 14-2t & \text{if } 5 \le t < 6 \\ 2 & \text{if } 6 \le t < t_{full} \end{cases}$$

- The function $r(t)$ above gives the rate of flow of hot water into Molly's tub in gallons per minute.

NOTE: The different intervals should tell you right away that your graph isn't going to be linear. Expect to see it increase & decrease at different rates.

{r(t) seen below}



- Water increases until it reaches 4 gallons of hot water per min.
- water fills tub at a constant rate for 3 min
- Someone in building starts Laundry. Flow of hot water decreases quickly.

2. What does the function $w(T) = \int_0^T r(t)\,dt$ for $T > 0$ represent? (What are its units?) {HINT: RECALL THE AREA PROBLEM DEF.}.

- When time is greater than zero ($T > 0$), $w(T)$ represents The total amount of water [under the line $r(t)$ and above the x-axis, in this case time (min)] within the interval $\int_0^T$.
  - ↪ The amount of water is measured in gallons per minute (gal/min)
  - ↪ The Time is measured in minutes (min)
  - ↪ The $\int_0^T$ represents the interval of time starting at 0, going T_{Full}.

⊛ NOTE : As T changes ⟶ $W(T)$ will also change!

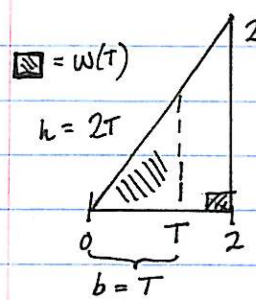
$\boxed{\text{IMPORTANT}}$: The goal here is to find out when (T_{Full}) Molly's bathtub will be full (contain 25 gallons) of water.

3. Use geometry to find expression of $w(T)$.

• Here you should use your graph from part 1 and consider doing a piecewise representation of the intervals given in $r(t)$. You should have 1 Shape for each interval and be able to use the given coordinates and or variables to determine it's area.

$$w(T) = \begin{cases} \text{i)}\ T^2 & \text{if } 0 \le T < 2 \\ \text{ii)}\ 4t - 4 & \text{if } 2 \le T < 5 \\ \text{iii)}\ -T^2 + 14t - 29 & \text{if } 5 \le T < 6 \\ \text{iv)}\ 2T + 7 & \text{if } 6 \le T < T_{FULL} \end{cases}$$

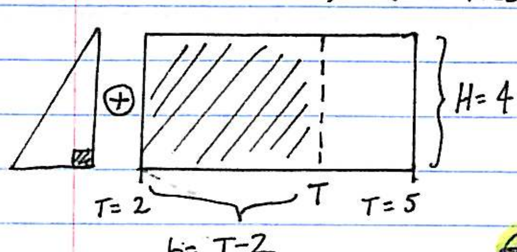
i) FIRST Consider interval $0 \le T < 2$ where $r(t) = 2T$
it's Shape is a triangle, where area is measured $A = \frac{bh}{2}$



$A\Delta = \frac{1}{2}(b)(h) \quad \therefore \quad A\Delta = \frac{1}{2}(T)(2T) = T^2$

*CHECK : T at 2 = 4 and $(2)^2 = 4$ ☑

ii) NEXT, Consider the long rectangle during interval $2 \le T < 5$ (3 min) where $r(t) = 4$. The Area can be measured by (B)(h) [base × height].

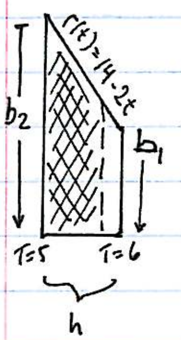


$A\square = (b)(h) \quad \therefore \quad A\square = (T-2)(4)$
$= 4t - 8$

BE MINDFUL NOT TO FORGET TO ADD THE $A\Delta$ AT 2 WHICH $= 4$. $\{\therefore A\square + \Delta = 4t - 4\}$

3. iii) NOW study the third interval where $5 \leq T < 6$ and $r(t) = 14 - 2t$. The shape is of a trapezoid who's area is measured by Taking $\frac{\text{height}}{2}(\text{base}_1 + \text{base}_2)$.

⊛ This interval is a little tricky. When identifying which sides are the bases$_{1\&2}$ and height know that the bases must be parrallel to each other (∥) and the height is not ⟂. (You might want practice to be sure which is which.



TRICK: TRY turning your head sideways!

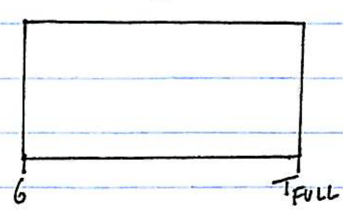
$$A = \frac{1}{2}(b_1 + b_2)(\text{height})$$
$$= \frac{1}{2}((14-2t)+4)(T-5) \longrightarrow \text{multiply out}$$
$$= -T^2 + 14t - 45$$

⊛ AGAIN: DON'T FORGET TO ADD THE AREAS OF THE PREVIOUS INTERVALS.

$$A\triangle + A\square + A = (-T^2 + 14T - 45) + 19$$
$$= -T^2 + 14T - 29$$

* Check: $T(6) = -(6)^2 + 14(6) - 29 = 19$ ☑

iv) FINALY, The last interval: $6 \leq T \leq T_{Full}$ where $r(t) = 2$ (constant) Assuming Molly has magical powers where she can stop the water from filling her bathtub at the snap of her fingers without any decrease in the rate of gallons per minute, The last shape will be a rectangle.



$$A = (B)(H) \therefore A = (T-6)(2)$$
$$A = 2T - 12 \longrightarrow \text{ADD TOTAL PREVIOUS AREA} = 19$$
$$A = 2T + 7$$

4. for interval $6 \leq T < T_{Full}$ $w(T) = 2T + 7$. if $2T + 7 = 25$ Then $T = 9$

$T_{FULL} = 9$ Which means that at 9 min, The tub will contain 25 gallons! ⟶