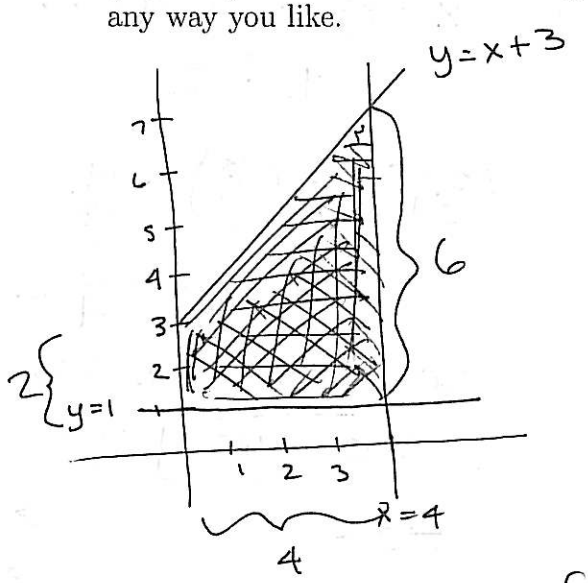
Math 2
February 6, 2008

Name: _____

Quiz 4

Show your work, and write clearly. No textbooks, notes, or calculators.

1. (3 points) Find the area of the region between $y = x + 3$, $y = 1$, $x = 0$, and $x = 4$, in any way you like.



First way: Area of a trapezoid

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$= \frac{1}{2}(6+2)(4) = \frac{1}{2} \cdot 8 \cdot 4 = 16$$

Second way: Integration

$$A = \int_0^4 (x+3) - (1)\,dx = \int_0^4 x+2\,dx$$

$$= \frac{x^2}{2} + 2x \Big|_0^4$$

$$= \left(\frac{4^2}{2} + 2(4)\right) - \left(\frac{0^2}{2} + 2(0)\right)$$

$$= \frac{16}{2} + 8 = 16$$

2. (7 points) Find the area of the region between $y = x^2 - 3x$ and $y = -2$ as follows:

(a) Find all points of intersection of these two graphs.

(b) Decide which curve's graph lies above the other.

(c) Write out the definite integral used to calculate area.

(d) Evaluate the definite integral to arrive at a numerical answer.

a) $x^2 - 3x = -2$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1, 2$

b) $x = \frac{3}{2}$ is between 1 & 2

$\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2}$

$= -\frac{9}{4} < -2$

so $y = x^2 - 3x$ is below $y = -2$

c) $A = \int_1^2 (-2) - (x^2 - 3x)\,dx$

$= \int_1^2 -2 - x^2 + 3x\,dx$

d) $= -2x - \frac{x^3}{3} + \frac{3x^2}{2}\Big|_1^2$

$= \left[-2(2) - \frac{(2)^3}{3} + \frac{3(2)^2}{2}\right] - \left[-2(1) - \frac{(1)^3}{3} + \frac{3(1)^2}{2}\right]$

$= \left[-4 - \frac{8}{3} + 6\right] - \left[-2 - \frac{1}{3} + \frac{3}{2}\right]$

$= \underbrace{(-4+6+2)}_{4} + \left(-\frac{8}{3} + \frac{1}{3}\right) - \frac{3}{2}$

$+ -\frac{7}{3} - \frac{3}{2} = \frac{24 - 14 - 9}{6} = \frac{1}{6}$