Math 2
January 18, 2008

Name: Solutions

Quiz 2

Show your work, and write clearly. No textbooks, notes, or calculators.

1. (3pts) Given $f(x) = \sqrt{x} + 3\cos x + 4$.

   (a) Find **all** antiderivatives of $f(x)$. Hint: Write $\sqrt{x}$ as a power of x.

   $f(x) = x^{1/2} + 3\cos x + 4$

   $F(x) = \frac{x^{3/2}}{3/2} + 3\sin x + 4x + C$

   (b) How can you check that your answer to part (a) is correct?

   Take the derivative of F...

   Is $F'(x) = f(x)$?

   (c) Check part (a) using the method from part (b).

   $F'(x) = \frac{3}{2}\left(\frac{x^{1/2}}{3/2}\right) + 3\cos x + 4 + 0$

   $= x^{1/2} + 3\cos x + 4$ ✓

2. (5pts) Suppose youre driving at 120 feet per second when you suddenly see a moose in the highway 250 feet ahead of you and you jam on the brakes, causing you to decelerate at 30 ft/s^2. Therefore, as you know, your velocity is $v(t) = 120 - 30t$, where t is the time in seconds since you saw the moose and hit the brakes, and v is in feet per second.

   (a) How long will it take for your car to reach a complete stop?

   (b) What is your stopping distance (the distance traveled during that time period)?

   (c) How close to the moose will you be when you come to a full stop?

   (d) Now, suppose youre tired from hunting, causing an additional reaction time of a quarter of a second, going along at your original steady 120 ft/sec, between when you see the moose and when you start the braking process described above. Will this delayed reaction time cause you to crash into the moose?

   (Hint: The only part that requires calculus is part (b))

a) need t when $v(t)=0$

$0 = 120 - 30t$

$30t = 120$

$t = 4s$

b) $s'(t) = v(t) \Rightarrow s(t) = 120t - 15t^2 + C$

$s(0) = 0 \Rightarrow C = 0$

$s(4) = 120(4) - 15(4^2)$

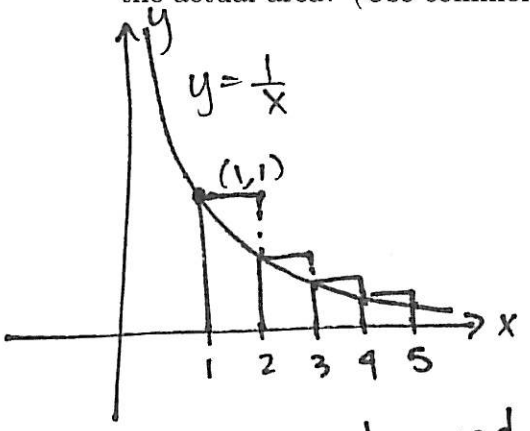
$= 480 - 240$

$= 240$ ft

c) $250 - 240 = 10$ ft

d) For .25s you travel $120\frac{ft}{s} \Rightarrow 30$ft extra, but you only have 10ft to spare... so yes the moose gets hit !!

3. (2pts) Approximate the area under $f(x) = 1/x$ from $x = 1$ to $x = 5$, using four rectangles of equal width, and left endpoints. Is this a lower bound or an upper bound for the actual area? (Use common denominators when you add.)



upper bound

$$L_4 = 1\left(\tfrac{1}{1}\right) + 1\left(\tfrac{1}{2}\right) + 1\left(\tfrac{1}{3}\right) + 1\left(\tfrac{1}{4}\right)$$

$$= 1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4}$$

$$= \frac{12+6+4+3}{12} = \frac{25}{12}$$

BONUS (2pts) "Therefore, as you know, your velocity is $v(t) = 120 - 30t$." As you know, do you know?

Yes, we do.

Velocity is the antiderivative of acceleration. and acceleration is constant, $-30\ \frac{ft}{s^2}$

So $v(t) = -30t + C$. When you start braking, at $t=0$, you're traveling $120\ \frac{ft}{s}$

$$v(0) = -30(0) + C = 120$$

$$\Rightarrow C = 120$$

$$\Rightarrow v(t) = 120 - 30t$$