

Quiz 1

Show your work, and write clearly. No textbooks, notes, or calculators.

1. Find the following limits, if they exist:

(a) $\lim_{x \rightarrow 0} \frac{|x|}{x} =$

(b) $\lim_{x \rightarrow \infty} \ln x =$

(c) $\lim_{n \rightarrow \infty} \left(\frac{81}{n^4} \left(\frac{n(n+1)}{2} \right)^2 - \frac{54}{n^2} \left(\frac{n(n+1)}{2} \right) \right) =$

2. Find the derivative of $f(\theta) = e^{\sin(\theta^2)}$.

3. Find the derivative of $y = \cos(2x)$ in two different ways.

(a) First, use the chain rule.

(b) Second, use the formula for cosine of a double angle: $\cos(2x) = \cos^2 x - \sin^2 x$.

(c) Check that your answers agree by using the formula for sine of a double angle: $\sin 2x = 2 \sin x \cos x$.

4. Suppose you leave Hanover right now by car, to get as far away from this quiz as possible. Your distance traveled (in miles) is given by $d(t) = 30t^2 + 20t$, where t is the time, in hours. The speed limit is 65 miles per hour, but the cops won't pull you over until you're going 80 miles per hour.

(a) Assuming that the area is heavily patrolled and that you'll be pulled over as soon as you hit 80 mph, how long will it be until you get pulled over?

(b) How many miles will you have traveled by then?

(c) Suppose now that you don't stop for the cops, and continue to flee the quiz as planned. How far away from Hanover will you be one hour after you see the flashing lights in your mirror?

(d) What will your speed be at this time?