MATH 2 : SOLUTIONS TO PROBLEM SET #2 :

REVIEW EXERCISES FOR CHAPTER #3 :

(2.) $y = \cos(\tan x)$

$\Rightarrow y' = -\sin(\tan x) \cdot \sec^2 x$ (CHAIN RULE)

(4.) $y = \dfrac{3x-2}{\sqrt{2x+1}}$

$$\Rightarrow y' = \frac{3\sqrt{2x+1} - (3x-2)\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{2x+1}}\cdot 2}{2x+1}$$

(QUOTIENT, CHAIN RULE)

$$= \frac{3\sqrt{2x+1} - \left(\frac{3x-2}{\sqrt{2x+1}}\right)}{2x+1}.$$

(8.) $y = e^{-t}(t^2 - 2t + 2)$

$\Rightarrow y' = e^{-t}(2t-2) - e^{-t}(t^2 - 2t + 2)$

(PRODUCT RULE)

$\Rightarrow y' = e^{-t}(2t - 2 - t^2 + 2t - 2)$

$= e^{-t}(-t^2 + 4t - 4)$

(SIMPLIFICATION).

(20.) $y = \ln(x^2 e^x)$

$\Rightarrow y' = \dfrac{1}{x^2 e^x}(2x e^x + x^2 e^x)$

(CHAIN & PRODUCT RULE)

$= \dfrac{x^2 + 2x}{x^2}$

(CANCEL OUT e^x)

$= 1 + \dfrac{2}{x}$ (SIMPLIFICATION, y NOT DEFINED AT $x = 0$ ANYHOW, SO OK.)

(22.) $y = \sec(1+x^2)$

$\Rightarrow y' = \sec(1+x^2)\tan(1+x^2)\cdot 2x$

(CHAIN RULE).

(32.) $y = e^{\cos x} + \cos(e^x)$

$\Rightarrow y' = e^{\cos x}(-\sin x) - \sin(e^x)\cdot e^x$

(SUM, CHAIN RULE).

(52.) $g(\theta) = \theta \sin\theta$

$\Rightarrow g'(\theta) = \sin\theta + \theta\cos\theta$ (PRODUCT RULE)

$\Rightarrow g''(\theta) = \cos\theta + \cos\theta - \theta\sin\theta$

(SUM, PRODUCT RULE.)

$= 2\cos\theta - \theta\sin\theta$

$\Rightarrow g''\left(\frac{\pi}{6}\right) = 2\left(\frac{\sqrt{3}}{2}\right) - \frac{\pi}{6}\left(\frac{1}{2}\right)$

$= \sqrt{3} - \frac{\pi}{12}$.

(58.) $y = \dfrac{x^2 - 1}{x^2 + 1}$, $(0, -1)$

$$y' = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2} \quad \text{(QUOTIENT RULE)}$$

$$= \frac{4x}{(x^2+1)^2}$$

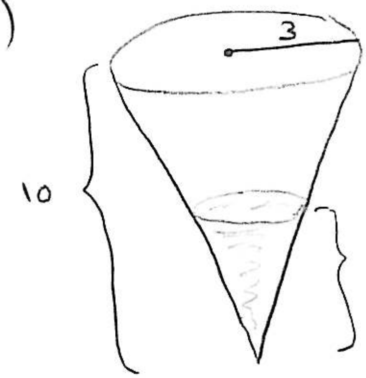
SO $y'(0) = 0$

SO THE TANGENT LINE TO $y = \dfrac{x^2-1}{x^2+1}$

AT $(0,-1)$ IS HORIZONTAL & THROUGH $(0,-1)$

SO IT HAS EQUATION $y = -1$.

(98.)



h = HEIGHT OF WATER AT TIME t

$$V = \frac{1}{3}\pi\left(\frac{3}{10}h\right)^2 h = \frac{3\pi}{100}h^3$$

GIVES THE VOLUME OF THE WATER AS A FUNCTION OF h.

$$\frac{dV}{dh}\cdot\frac{dh}{dt} = \frac{dV}{dt} = 2$$

NOW, $$\frac{dV}{dh} = \frac{9\pi}{100}h^2$$

SO $$\frac{9\pi}{100}h^2\cdot\frac{dh}{dt} = 2$$

WHEN $h = 5$, GET

$$\frac{9\pi}{4}\cdot\frac{dh}{dt} = 2$$

SO $$\frac{dh}{dt} = \frac{8}{9\pi} \text{ cm/sec.}$$