MATH 2 SOLUTIONS TO PROBLEM SET # 18

SECTION 7.5 - STRATEGY FOR INTEGRATION

(1.) $\int \cos x\,(1+\sin^2 x)\,dx = \int 1+u^2\,du = u+\frac{1}{3}u^3 + C$

$= \boxed{\sin x + \frac{1}{3}\sin^3 x + C}$.

SUBSTITUTION (5.5):

$u = \sin x$

$du = \cos x\,dx$

(CHECK BY DIFFERENTIATION, AS ALWAYS.)

(4.) $\int \tan^3\theta\,d\theta = \int \tan\theta \cdot \tan^2\theta\,d\theta$

$= \int \tan\theta\,(\sec^2\theta - 1)\,d\theta$

$= \int \tan\theta\,\sec^2\theta\,d\theta - \int \tan\theta\,d\theta$

1ST INTEGRAL: SUBSTITUTION (5.5)

$u = \tan\theta$

$du = \sec^2\theta\,d\theta$

2ND INTEGRAL: SUBSTITUTION (5.5)

$w = \cos\theta$

$dw = -\sin\theta\,d\theta$

$= \int u\,du - \int -\frac{1}{w}\,dw = \int u\,du + \int \frac{1}{w}\,dw$

$= \frac{u^2}{2} + \ln|w| + C = \boxed{\frac{\tan^2\theta}{2} + \ln|\cos\theta| + C}$.

CHECK: $\frac{d}{d\theta}\left(\frac{\tan^2\theta}{2} + \ln|\cos\theta|\right) = \tan\theta\sec^2\theta - \frac{\sin\theta}{\cos\theta}$

$= \tan\theta\sec^2\theta - \tan\theta = \tan\theta(\sec^2\theta - 1) = \tan^3\theta$. ✓

(5) $\displaystyle\int_0^2 \frac{2t}{(t-3)^2}\,dt = \int_0^2 \frac{2}{t-3} + \frac{6}{(t-3)^2}\,dt$

$\displaystyle = \int_0^2 \frac{2}{t-3}\,dt + \int_0^2 \frac{6}{(t-3)^2}\,dt$

$\displaystyle = \Big[\,2\ln|t-3|\,\Big]_0^2 + \Big[\,\frac{-6}{t-3}\,\Big]_0^2$

$u = t-3$
$du = dt$

$= \underbrace{2\ln(1)}_{0} - 2\ln(3) + 6 - 2$

$= \boxed{4 - 2\ln(3)}$.

PARTIAL FRACTIONS (7.4):

$\displaystyle \frac{2t}{(t-3)^2} = \frac{A}{t-3} + \frac{B}{(t-3)^2}$

$2t = A(t-3) + B$

$\begin{cases} A = 2 \\ B - 3A = 0 \end{cases}$

So $A=2,\ B=6$

$\displaystyle \frac{2t}{(t-3)^2} = \frac{2}{t-3} + \frac{6}{(t-3)^2}$

(15.) $\displaystyle\int \frac{dx}{(1-x^2)^{3/2}} = \int \frac{\cos\theta}{\cos^3\theta}\,d\theta = \int \sec^2\theta\,d\theta = \tan\theta + C$

$\displaystyle = \frac{\sin\theta}{\cos\theta} + C = \boxed{\frac{x}{\sqrt{1-x^2}} + C}$.

TRIGONOMETRIC SUBSTITUTION (7.3):

$x = \sin\theta$
$1-x^2 = \cos^2\theta$
$dx = \cos\theta\,d\theta$

(19.) $\displaystyle\int e^{x+e^x}\,dx = \int e^x e^{e^x}\,dx = \int e^u\,du$

$= e^u + C = \boxed{e^{e^x} + C}$.

SUBSTITUTION (5.5):

$u = e^x$
$du = e^x\,dx$

(24.) $\displaystyle\int \ln(x^2-1)\,dx = \int \ln((x+1)(x-1))\,dx = \int \ln(x+1) + \ln(x-1)\,dx$

$\displaystyle = \int \ln(x+1)\,dx + \int \ln(x-1)\,dx.$

BY SUBSTITUTION, THIS IS $F(x+1) + F(x-1) + C$, WHERE $F(x)$ IS AN ANTIDERIVATIVE OF $\ln x$.

INTEGRATION BY PARTS (7.1): $\displaystyle\int \ln x\,dx = x\ln x - \int dx = x\ln x - x + C.$

$u = \ln x \quad v = x$
$dv = dx \quad du = \frac{1}{x}\,dx$

THUS TAKE $F(x) = x\ln x - x + C$.

SO THE ANSWER IS $(x+1)\ln(x+1) - (x+1) + (x-1)\ln(x-1) - (x-1) + C$

$= \boxed{(x+1)\ln(x+1) + (x-1)\ln(x-1) - 2x + C}$.