MATH 2 — SOLUTIONS TO PROBLEM SET #15

SECTION 7.2 - TRIGONOMETRIC INTEGRALS

(1.) $\int \sin^3 x \cos^2 x \, dx = \int \sin^2 x \cos^2 x \cdot \sin x \, dx$

$= \int (1-\cos^2 x) \cos^2 x \sin x \, dx = \int (\cos^2 x - 1) \cos^2 x \cdot -\sin x \, dx$

$\boxed{\begin{array}{l} u = \cos x \\ du = -\sin x \, dx \end{array}}$

$= \int (u^2 - 1) u^2 \, du = \int u^4 - u^2 \, du$

$= \frac{1}{5} u^5 - \frac{1}{3} u^3 + C = \frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C$

$= \boxed{\cos^3 x \left(\frac{1}{5} \cos^2 x - \frac{1}{3} \right) + C}.$

AS ALWAYS, YOU CAN CHECK BY DIFFERENTIATION.

(2.) $\int \sin^6 x \cos^3 x \, dx = \int \sin^6 x \cos^2 x \cdot \cos x \, dx$

$= \int \sin^6 x (1 - \sin^2 x) \cdot \cos x \, dx = \int u^6 (1 - u^2) \, du$

$\boxed{\begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array}}$

$= \int u^6 - u^8 \, du = \frac{u^7}{7} - \frac{u^9}{9} + C$

$= \frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C = \boxed{\sin^7 x \left(\frac{1}{7} - \frac{\sin^2 x}{9} \right) + C}.$

CHECK: $\frac{d}{dx} \left(\frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} \right) = \sin^6 x \cos x - \sin^8 x \cos x$

$= \cos x \cdot \sin^6 x (1 - \sin^2 x) = \cos x \cdot \sin^6 x \cdot \cos^2 x$

$= \sin^6 x \cos^3 x$. ✓

(3.) $\displaystyle\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x \, dx = \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \sin^5 x (1-\sin^2 x)\cos x \, dx$

$\boxed{\begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array}}$ $\displaystyle = \int_1^{\frac{1}{\sqrt{2}}} u^5 (1-u^2)\, du = \int_1^{\frac{1}{\sqrt{2}}} u^5 - u^7 \, du$

$$= \left[\frac{u^6}{6} - \frac{u^8}{8}\right]_1^{\frac{1}{\sqrt{2}}} = \left[\left(\frac{1}{48} - \frac{1}{128}\right) - \left(\frac{1}{6} - \frac{1}{8}\right)\right]$$

$$= \frac{8}{384} - \frac{3}{384} - \frac{64}{384} + \frac{48}{384} = \boxed{-\frac{11}{384}}.$$

$\boxed{(4.)}$ $\displaystyle\int_0^{\pi/2} \cos^5 x \, dx = \int_0^{\pi/2} \cos^4 x \cdot \cos x \, dx$

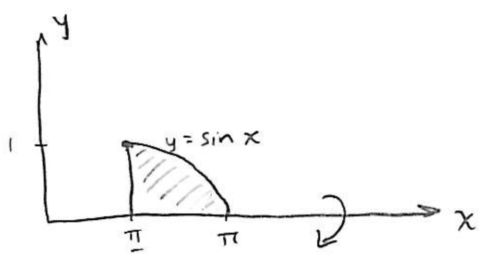
$$= \int_0^{\pi/2} (1-\sin^2 x)^2 \cos x \, dx = \int_0^1 (1-u^2)^2 \, du$$

$\boxed{\begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array}}$ $\displaystyle = \int_0^1 u^4 - 2u^2 + 1 \quad du$

$$= \left[\frac{u^5}{5} - \frac{2u^3}{3} + u\right]_0^1 = \frac{1}{5} - \frac{2}{3} + 1 = \boxed{\frac{8}{15}}.$$

(61.) $y = \sin x$, $y = 0$, $x = \frac{\pi}{2}$, $x = \pi$; x-AXIS

BY DISKS, $\displaystyle V = \pi \int_{\frac{\pi}{2}}^{\pi} \sin^2 x \, dx$



$$= \pi \left[\frac{x}{2} - \frac{\sin(2x)}{4}\right]_{\frac{\pi}{2}}^{\pi}$$

$$= \pi\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \pi\left(\frac{\pi}{4}\right) = \boxed{\frac{\pi^2}{4}}.$$

(65.) $v(t) = \sin \omega t \cos^2 \omega t$

$f(t)$ IS AN ANTIDERIVATIVE OF $v(t)$, SO IT'S OF THE FORM

$$\int \sin \omega t \cos^2 \omega t \, dt = -\frac{1}{\omega} \int \cos^2 \omega t \cdot -\omega \sin \omega t \, dt$$

$$\boxed{\begin{aligned} u &= \cos \omega t \\ du &= -\omega \sin \omega t \, dt \end{aligned}} \quad = -\frac{1}{\omega} \int u^2 \, du = -\frac{1}{\omega}\left[\frac{u^3}{3}\right] + C$$

$$= -\frac{1}{3\omega} \cos^3 \omega t + C,$$

NOW, $f(0) = 0 \Rightarrow -\frac{1}{3\omega} + C = 0 \Rightarrow C = \frac{1}{3\omega}$.

THUS $f(t) = -\frac{1}{3\omega} \cos^3 \omega t + \frac{1}{3\omega} = \boxed{\frac{1}{3\omega}\left(1 - \cos^3 \omega t\right)}$.