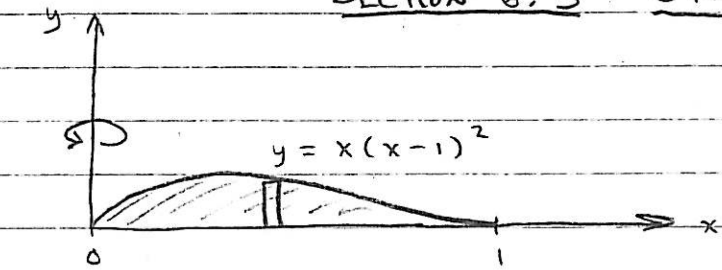
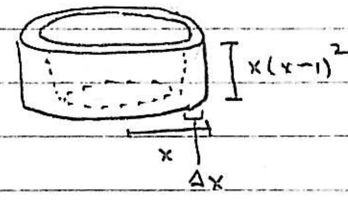
MATH 2 SOLUTIONS TO PROBLEM SET #11

SECTION 6.3 : CYLINDRICAL SHELLS

(1.)





TYPICAL SHELL

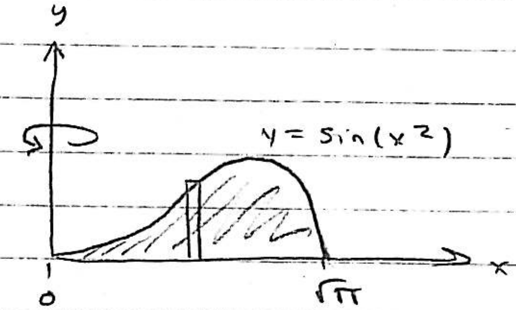
$$V = 2\pi \int_0^1 x \cdot x(x-1)^2 \, dx$$

$$= 2\pi \int_0^1 (x^4 - 2x^3 + x^2) \, dx$$

$$= 2\pi \left[\frac{1}{5}x^5 - \frac{1}{2}x^4 + \frac{1}{3}x^3 \right]_0^1 = 2\pi\left(\frac{1}{30} - 0\right) = \boxed{\frac{\pi}{15}}$$

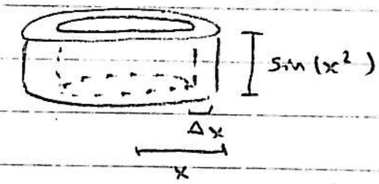
FOR THIS PROBLEM, THE CYLINDRICAL SHELLS METHOD IS FAR MORE PREFERABLE THAN SLICING.
FOR SLICING, WE'D HAVE TO SOLVE FOR x AS A FUNCTION OF y, FIND THE MAXIMUM VALUE a OF $x(x-1)^2$ ON $[0,1]$, AND THEN INTEGRATE THIS FUNCTION OF y ON $[0,a]$.
SOLVING CUBIC EQUATIONS IS MUCH HARDER AND MESSIER THAN SOLVING QUADRATICS.

$\boxed{2.}$



TYPICAL SHELL

$V = 2\pi x \sin(x^2)\, \Delta x$



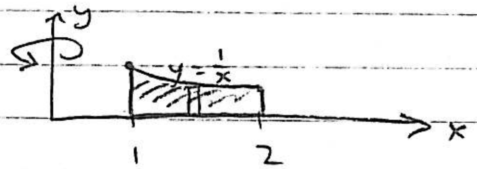
$$V = \pi \int_0^{\sqrt{\pi}} 2x \cdot \sin(x^2)\, dx = \pi \int_0^{\pi} \sin(u)\, du$$

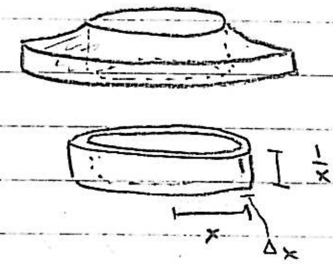
$$\boxed{\begin{aligned} \text{LET } u &= x^2 \\ du &= 2x\, dx \end{aligned}} \qquad = \pi \left[-\cos u\right]_0^{\pi}$$

$$= \pi(1+1) = \boxed{2\pi}.$$

FOR THIS PROBLEM, CYLINDRICAL SHELLS IS FAR MORE PREFERABLE THAN SLICING. THE ADDITIONAL X-TERM IN THE INTEGRAND ACTUALLY MAKES THE INTEGRAL MUCH SIMPLER, SINCE IT BECOMES POSSIBLE TO USE SUBSTITUTION. FOR SLICING, WE'D HAVE TO SOLVE $y = \sin(x^2)$ FOR x, FIND THE MAXIMUM VALUE OF $\sin(x^2)$ ON $[0, \sqrt{\pi}]$, CALL IT a, AND THEN INTEGRATE THIS FUNCTION ON $[0, a]$.

(3.) $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 2$; ABOUT THE y-AXIS

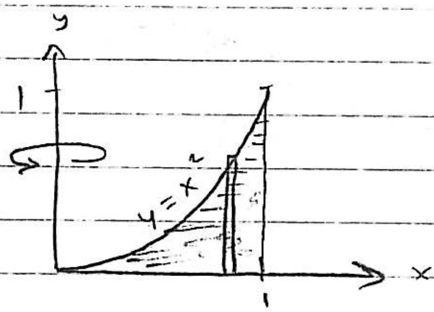


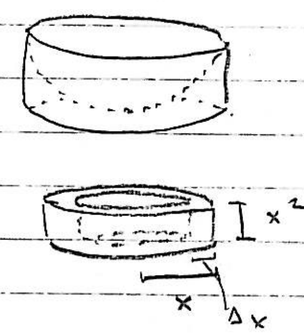


$$V = 2\pi \int_1^2 x \cdot \frac{1}{x} \, dx = 2\pi \int_1^2 dx$$

$$= 2\pi \left[x \right]_1^2 = 2\pi(2-1) = \boxed{2\pi}.$$

$\boxed{(4.)}$ $y = x^2$, $y = 0$, $x = 1$; ABOUT THE y-AXIS

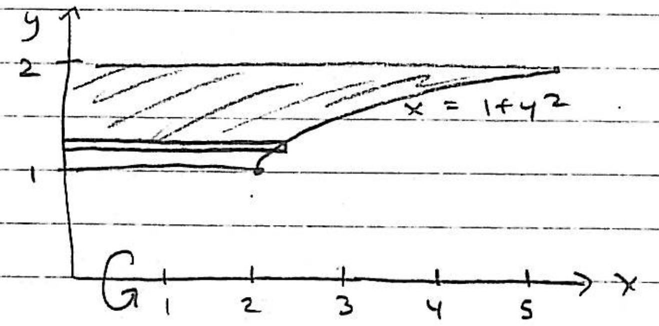




$$V = 2\pi \int_0^1 x \cdot x^2 \, dx = 2\pi \int_0^1 x^3 \, dx$$

$$= 2\pi \left[\frac{x^4}{4} \right]_0^1 = 2\pi \left(\frac{1}{4} - 0 \right) = \boxed{\frac{\pi}{2}}.$$

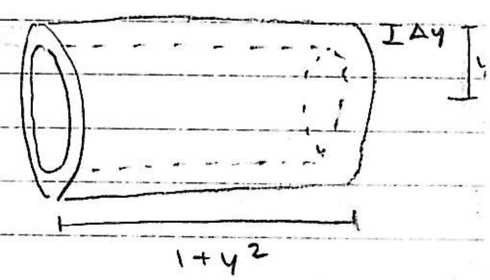
(9.) $x = 1 + y^2$, $x = 0$, $y = 1$, $y = 2$; X-AXIS



$$V = 2\pi \int_1^2 y\,(1+y^2)\,dy = 2\pi \int_1^2 (y + y^3)\,dy$$

$$= 2\pi \left[\frac{y^2}{2} + \frac{y^4}{4} \right]_1^2 = 2\pi \left((2+4) - \frac{3}{4} \right)$$

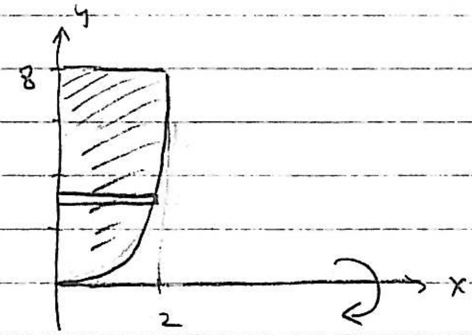
$$= 2\pi \left(\frac{21}{4} \right) = \boxed{\frac{21\pi}{2}}.$$

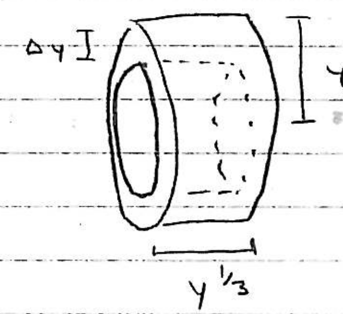


(11.) $y = x^3$, $y = 8$, $x = 0$; X-AXIS

$\Updownarrow$

$x = y^{1/3}$





$$V = 2\pi \int_0^8 y \cdot y^{\frac{1}{3}}\, dy$$

$$= 2\pi \int_0^8 y^{4/3}\, dy = 2\pi \left[\frac{3}{7} y^{\frac{7}{3}} \right]_0^8$$

$$= 2\pi \left(\frac{3}{7} \cdot 2^7 - 0 \right) = \frac{3\pi}{7} \cdot 256 = \boxed{\frac{768\pi}{7}}.$$