Math 2
March 7, 2008

Name: ____________________

Extra Quiz

Show your work, and write clearly. No textbooks, notes, or calculators. Use which ever methods you would like to evaluate the following integrals.
Some trig identities you may find helpful:

$\cos(2x) = \cos^2 x - \sin^2 x$	$\sin(2x) = 2\sin x\cos x$
$\sin^2 x = \frac{1-\cos(2x)}{2}$	$\cos^2 x = \frac{1+\cos(2x)}{2}$

1. $\int_0^4 \frac{x-1}{x^2-4x-5}\,dx = \int_0^4 \frac{x-1}{(x-5)(x+1)}\,dx = \int_0^4 \frac{A}{x-5} + \frac{B}{x+1}\,dx$

$= \int_0^4 \frac{A(x+1)+B(x-5)}{(x-5)(x+1)}\,dx = \int_0^4 \frac{(A+B)x + A - 5B}{(x-5)(x+1)}\,dx$

$A+B=1$

$A-5B=-1$

$A = \quad = \frac{2}{3}$

$1-B-5B=-1$

$-6B=-2$

$B=\frac{1}{3}$

$= \int_0^4 \frac{\frac{2}{3}}{x-5} + \frac{\frac{1}{3}}{x+1}\,dx$

$= \frac{2}{3}\ln|x-5| + \frac{1}{3}\ln|x+1| \Big]_0^4$

$= \cancel{\frac{2}{3}\ln|-1|} + \frac{1}{3}\ln|5| - \frac{2}{3}\ln|-5| + \cancel{\frac{1}{3}\ln|1|}$

$= \frac{1}{3}\ln 5 - \frac{2}{3}\ln 5 = -\frac{1}{3}\ln 5$

2. $\int_0^{\frac{\pi}{2}} \sin^3\theta \cos^2\theta \, d\theta \;=\; \int_0^{\pi/2} \sin^2\theta \cos^2\theta \; \sin\theta \, d\theta$

$= \int_0^{\pi/2} (1-\cos^2\theta)\cos^2\theta \; \sin\theta \, d\theta$

$u = \cos\theta$

$du = \sin\theta \, d\theta$

$0 \longrightarrow \cos(0) = 1$

$\frac{\pi}{2} \longrightarrow \cos\frac{\pi}{2} = 0$

$= \int_1^0 (1-u^2)u^2(-du) \;=\; \int_0^1 u^2 - u^4 \, du$

$= \frac{u^3}{3} - \frac{u^5}{5} \Big|_0^1 = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

3. $\int \frac{x+1}{\sqrt{3-2x-x^2}} \, dx \;=\; -\frac{1}{2} \int \frac{1}{\sqrt{u}} \, du = -\frac{1}{2} \int u^{-1/2} \, du$

$= -\frac{1}{2} \frac{u^{1/2}}{1/2} + C \;=\; -\sqrt{3-2x-x^2} + C$

$u = 3-2x-x^2$

$du = -2-2x \, dx$

$= -2(1+x)dx$

$x+1 \, dx = -\frac{du}{2}$