Math 2 Due: February 15, 2008

Extra Credit

This is a written assignment for a possible 5 points of extra credit that may be applied to your first exam score. On this week's assignment you should have needed to integrate the function $f(x) = \sin^2 x$. We will use this integral as the starting point for this assignment. The following steps are just to lead you through a process of discovery. The purpose of this assignment is to better understand the substitution method of solving integrals. Going through all these steps should give you a better idea of how the method works, how to apply it and why it makes sense in a visual/area way. Your final paper that you turn in should not be a list of the steps and how you worked them out. It should be a clear write up, perhaps including all the steps and definitely including your own reflection on the steps. Does this problem help you better understand substitution? Why does substitution work? It should make sense as a picture and as a formula, explain this.

Make sure you organize your thoughts well, and remember this is **extra credit**, so it will be graded more difficultly than regular weekly assignments. Good Luck!

- Start by considering the definite integral

$$\int_0^\pi \sin^2 x \, dx$$

- Rewrite the integral using an identity from trigonometry:

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

- Factor the constant outside the integral. We haven't changed the variable yet, so graph the new function on the xy-plane, from 0 to π.
- Because of the "function inside another function" scenario, we should use substitution to solve. Write down the substitution you would use, and how the new u-integral should look, including the new limits of integration.
- Again factor out the constant, this should be the one you got from changing dx to du, and draw a graph of the function you are integrating on the uy-plane.
- You should now have two drawings, one in the xy-plane and one in the uy-plane. Show by shading and/or labeling how the integrals represent area or net area.
- There is a lot going on here: stretching, scaling, constant factoring in and out, etc. Does it make sense that u-substitution should give you a correct answer? Why? How? Explain!