NAME:_____

MATH 2 EXAM 2

February 21, 2008

INSTRUCTIONS: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- Print your name clearly in the space provided.
- You may not use a calculator.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

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Signature

Question	Points	Score
1	20	
2	20	
3	20	
4	10	
5	10	
6	10	
7	10	
8	0	
Total:	100	

1. [20 points] Find the area of the region in the plane bounded by the graphs of $y = 2x + 1$ and $y = 2x^3 + 1$.
   (Hint: This region has two parts.)

2. (a) [10 points] Find the average value of $f(x) = \frac{1}{x}$ on the interval $[2, 6]$.

(b) [10 points] For what number(s) c in this interval is the average value you just found actually attained, i.e. $f(c) = f_{\text{ave}}$?

3. [20 points] Suppose that it takes a force of 10 pounds to hold a certain spring 6 inches past its natural length. How much work, in foot-pounds, is required to stretch this spring from its natural length to 2 feet past its natural length?
(Hint: By Hookes Law, the force it takes to hold a spring stretched a distance x past its natural length is proportional to x.)

4. [10 points] Find the volume of a solid sphere of radius r. Your answer should be in terms of r.
   (Hint: This is the solid of revolution obtained by revolving the region between $y = 0$ and $y = \sqrt{r^2 - x^2}$ about the x-axis.)

5. Find the volume of the solid of revolution obtained by revolving the region between the graphs of $y = 3x$ and $y = 3\sqrt{x}$ about the x-axis. Do so in two ways, to hopefully arrive at the same answer.

   (a) [5 points] Use washers:

   (b) [5 points] Use shells:

6. Find the volume of the solid of revolution obtained by revolving the same region in #5, but now about the y-axis. Do so in two ways, hopefully arriving at the same answer, which is , however, smaller than your answer to #5.

   (a) [5 points] Use washers:

   (b) [5 points] Use shells:

7. [10 points] Use a method of your choice to find the volume of the solid of revolution obtained by revolving the region between the curves $y = 3x\sin(x^3) + 5$, $y = 0$, $x = 0$, and $x = \sqrt[3]{\pi}$, about the y-axis.

8. **Bonus Problems** (4 points each)

   (a) What theorem guarantees that at least one such number c exists in 2(b)? Write down this theorem, including **all** the hypotheses, and also prove it, using the mean value theorem for derivatives and the Fundamental Theorem of Calculus.

(b) Let $f(t) = t\sin(t^2)$, and let $g(x)$ be the average value of $f(t)$ from $t = 0$ to $t = x$. What is $g'(x)$? Your answer should be in terms of x only.