Math 2

Sample Exam

1. At the Maynard Street construction site, there is a cone-shaped pile of gravel. A conveyer belt is positioned so that 3 cubic feet of gravel fall off of the conveyer belt onto the top of the pile every minute. Suppose that the ratio between the height h and the radius r of the cone remains constant, with $r = \frac{1}{3}h$.

   (Remember: In this problem, all variables are functions of time.)

   (a) Give an equation that relates the rate of change of the radius to the rate of change of the height of the cone.

   (b) Suppose that at a certain moment, the height of the cone is increasing at a rate of 2 feet per minute. What is the rate of change of the radius at this moment?

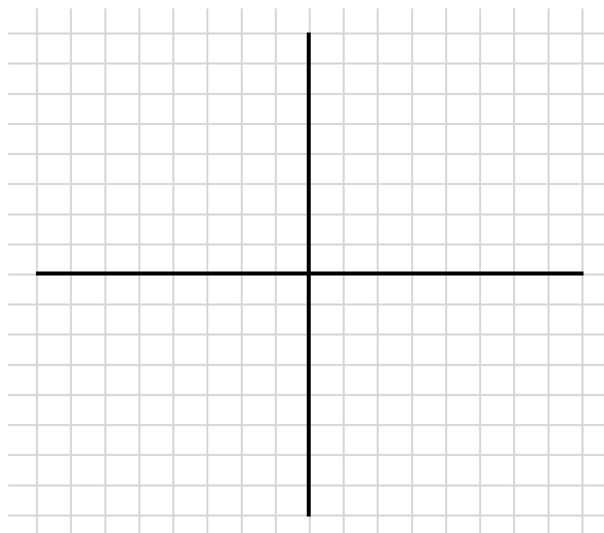
(c) At what rate is the height of the cone increasing when height is equal to 9 ft.?

(*Hint*: The volume of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

2. $f(x)$ is a piecewise function defined on the interval $[-3, 5]$ as below:

$$f(x) = \begin{cases} -x+1 & \text{if } -3 \le x \le -1 \\ 2x+4 & \text{if } -1 \le x \le 5 \end{cases}$$

(a) Using the coordinate axes given below, draw a graph of $f(x)$.



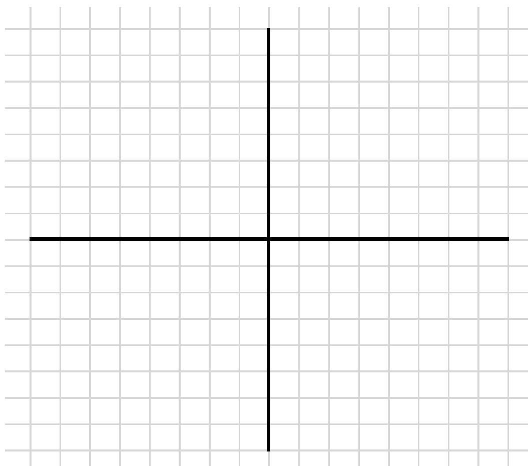
(b) What is the value of $\int_{-3}^{5} f(x)dx$?

(c) What is the average value of $f(x)$ on the interval $[-3, 5]$?

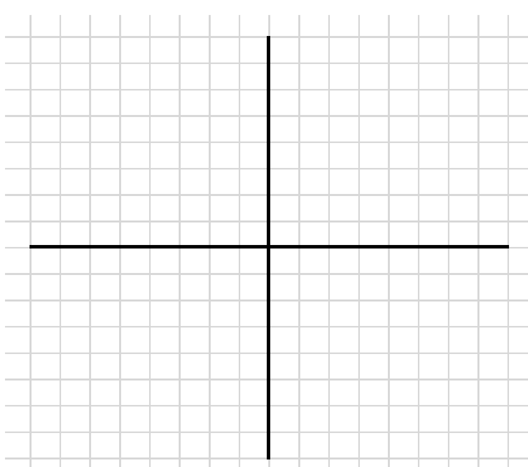
3. Given a function, $f(x)$, defined on the interval $[-2, 8]$, how do we define the definite integral of $f(x)$ from -2 to 8: $\int_{-2}^{8} f(x)dx$?

4. For parts (a) through (c), draw a graph of the given function, then give the value of requested integral. (*Note*: You may use any scaling you wish on these graphs, but be explicit.)

(a) $f(x) = \frac{1}{2}x - 2$ $\int_0^6 \left(\frac{1}{2}x - 2\right)\,dx = ?$



(b) $g(x) = \sin(x)$ $\int_{-\pi/2}^{\pi/2} \sin(x)\,dx = ?$



(c) $h(x) = \sqrt{25 - x^2}$ $\int_0^5 \sqrt{25 - x^2}\, dx = ?$