Term Paper Topic Ideas

Note that depending on the topic and specific example(s) chosen, a single example might not be sufficient for a whole paper. For example, you might need to discuss several modifications of Turing machines to get in the neighborhood of 8 pages.

This list is meant to suggest, not encompass. Please feel free to look outside its bounds.

(1) Describe another model of computation equivalent to Turing machines and prove the equivalence. Examples: Herbrand-Gödel computability, Post production (canonical) systems, modifications of Turing machines (nondeterminism, multiple tapes), matrix grammars, Markov algorithms, L-systems (Lindenmayer).

(2) Describe the implementation of a Turing machine in cellular automata including Conway's Game of Life or Wolfram's Rule 110.

(3) Describe the restrictions put on computability by (some flavor of) the philosophy of intuitionism (or constructivism). Give their justification and prove some mathematical consequences.

(4) Describe another undecidable problem and prove it is undecidable. Examples: word problem for (semi)groups, number theory, matrix mortality problem, tiling the plane with Wang tiles, isomorphism problem for groups, program validation.

(5) Discuss a couple of *strong reducibilities*; that is, reductions r such that $A \leq_r B \rightarrow A \leq_T B$ but not vice-versa. Prove some facts about the structure of degrees and/or their relationship with Turing and other reducibilities. Examples: many-one and 1-1 (as used in the arithmetic hierarchy material), truth table and weak truth table.

(6) Give alternate constructions of (possibly c.e.) noncomputable incomplete sets; for example Kleene-Post (not c.e.) and Friedberg-Muchnik (c.e.; introduction of the priority method).

(7) Prove and discuss the Friedberg and Owings Splitting Theorems, given in the book as Theorems 9.1.13 and 9.1.14.

(8) Define hypersimplicity and prove some basic results, such as that all h-simple sets are simple (Definition 5.4.1), there is an alternate characterization in terms of majorization, and every c.e. degree is h-simple. If that is insufficient add some results about hyperhypersimplicity.

(9) Define Π^0_1 classes (each is the set of infinite paths through some computable binary tree, possibly with dead ends) and give some constructions of classes representing specific objects such as ideals of rings, and some basic results such as the Low Basis Theorem.

(10) Discuss one or more approaches to computability in the real numbers: computable analysis, Blum-Shub-Smale (BSS) machines.

(11) Explore the d.c.e. sets, sets that can be written as $A - B$ for c.e. sets A and B: show non-c.e. examples exist, show in fact an example not of c.e. degree exists, or go into n-c.e. sets as a generalization and show the containment of n-c.e. in $n+1$-c.e. is proper.

(12) Define creative and productive sets and prove creative sets are all complete but not simple; show the index set $A_x = \{y : \phi_y = \phi_x\}$ is productive for each x; discuss the connection to automorphisms of the lattice of c.e. sets.