

## A False Induction Proof

For use in Homework #5, Math 25, Fall 2006

Consider the following claim, and its purported proof.

**Claim:** Let  $n \geq 1$ . Then, in any group of  $n$  people, everyone in that group has the same age.

*“Proof” of Claim :*

1. Base Case: In any group that consists of just one person, everybody in the group has the same age (trivially). Therefore, the claim is true for  $n = 1$ .
2. Inductive Hypothesis: assume that for every group of  $n$  people, everyone has the same age.
3. Let  $G$  be an arbitrary group of  $n + 1$  people; we need to show that every member of  $G$  has the same age.
4. To this end, let  $P$  and  $Q$  be any members of  $G$ ; we want to show that  $P$  and  $Q$  have the same age.
5. Consider everybody in  $G$  except for  $P$ . These people form a group of  $n$  people, so they must all have the same age (by the inductive hypothesis).
6. Now consider everybody in  $G$  except for  $Q$ . Again, this forms a group of  $n$  people, so they must all have the same age.
7. Now let  $R$  be someone else in  $G$  other than  $P$  or  $Q$ .
8. Since  $Q$  and  $R$  each belong to the group considered in step 5, they are the same age.
9. Since  $P$  and  $R$  each belong to the group considered in step 6, they are the same age.
10. Since  $Q$  and  $R$  are the same age, and  $P$  and  $R$  are the same age, it follows that  $P$  and  $Q$  are the same age.
11. Thus, any two people  $P$  and  $Q$  in the group  $G$  have the same age. It follows that everyone in  $G$  has the same age.
12. Therefore, everyone in any group of  $n$  people has the same age.

**Exc. S1:** The conclusion is clearly erroneous. Which step is the faulty one? Explain why.