

Math 24
Winter 2010
Special Assignment due Wednesday, January 20

Please hand in this assignment on a different piece of paper than the regular homework. Do not staple it together with the regular homework. (Be sure your name is on all assignments.)

Let V be any vector space and W be a subspace of V . For any vector x in V , we define the *coset* of W containing x to be

$$x + W = \{x + w \mid w \in W\}.$$

For example, if $V = \mathbb{R}^2$ and W is the x -axis, then

$$(1, 1) + W = \{(1, 1) + (x, y) \mid (x, y) \in W\} = \{(1, 1) + (x, 0) \mid x \in \mathbb{R}\} = \{(x + 1, 1) \mid x \in \mathbb{R}\}.$$

That is, $(1, 1) + W$ is the line $y = 1$. In general, if W is the x -axis, the coset of W containing (x, y) is the line through (x, y) parallel to the x -axis.

Assignment: Prove the following. For any vector space V and any subspace W of V .

1. For any x in V ,

$$x \in (x + W).$$

2. For any x and y in V ,

$$(x - y) \in W \implies (x + W) = (y + W).$$

3. For any x and y in V ,

$$(x - y) \notin W \implies (x + W) \cap (y + W) = \emptyset.$$

Note: What you have shown is that the cosets of W form a *partition* of V . This means that different (unequal) cosets are disjoint (do not overlap), and together, the cosets cover all of V . In the example, the lines parallel to the x -axis form a partition of \mathbb{R}^2 .

Note: If X and Y are sets, one way to prove $X = Y$ is to prove every element of X is in Y , and every element of Y is in X . One way to prove $X = \emptyset$ is to assume there is a element in X , and get a contradiction.