Math 24

Winter 2010

Wednesday, February 10

(1.) TRUE or FALSE?

(a.) Any system of linear equations has at least one solution.

(b.) Any system of linear equations has at most one solution.

(c.) Any homogeneous system of linear equations has at least one solution.

(d.) Any system of n linear equations in n unknowns has at most one solution.

(e.) Any system of n linear equations in n unknowns has at least one solution.

(f.) If the homogenous system corresponding to a given system of linear equations has a solution, then the given system has a solution.

(g.) If the coefficient matrix of a homogeneous system of n linear equations in n unknowns is invertible, then the system has no nonzero solutions.

(h.) The solution set of any system of m linear equations in n unknowns is a subspace of F^n.

(2.) Let A denote the coefficient matrix of the following system of linear equations. Compute A^{-1}, and use A^{-1} to solve the system.

$$x + 2y - z = 5$$
$$x + y + z = 1$$
$$2x - 2y + z = 4$$

(3.) Find the set of solutions for the homogeneous system associated to the following system of linear equations, and determine whether this system has a solution.

$$x + 2y - z = 1$$
$$2x + y + 2z = 3$$
$$x - 4y + 7z = 4$$

(4.) Find the set of solutions for the following system of linear equations. Note that the coefficient matrix is the same as in problem (3), and one obvious solution is $x = 1$, $y = z = 0$.

$$x + 2y - z = 1$$

$$2x + y + 2z = 2$$

$$x - 4y + 7z = 1$$

(5.) Prove or give a counterexample to the following statement: If the coefficient matrix of a system of m linear equations in n unknowns has rank m, then the system has a solution.

(6.) Prove or give a counterexample to the following statement: If $m < n$, then a system of m linear equations in n unknowns (over the field $\mathbb{R}$) has either no solutions or infinitely many solutions.